Name: AREMU BLESSING AYOMIDE
Department: PHARMACY
Matric No: 19/MHS11/033
Course code: MAT104

1) $y = \frac{1}{x-2}$

Ans

- The function is defined for all real numbers except $x = 2$
- The domain is the set of real numbers except $x = 2$
- The codomain of the set of real number except $y = 0$

2) $k = \ln V$

$\frac{dk}{dv} = \frac{1}{V}$

3) a) $2x - 3y - 2 = 0$

Sol

$2x - 3y - 2 = 0$

$\frac{-3y}{-3} = \frac{\cancel{-2x} + 2}{\cancel{-3y}}$

$y = \frac{2x+2}{3} : \frac{2(x+1)}{3}$

b) $x^2 + y^2 = 4$

Sol

$x^2 + y^2 = 4$

$\cancel{y^2 = x^2 - 4}$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

4) ~~Find~~ If $P = \sin^{-1} t$, find the derivative of $p$.

Sol

let $P = \sin^{-1} t$

$P = \frac{t}{\sin}$ ; $t = \sin p$

$\frac{dt}{dp} = \cos p$ ; $\frac{dp}{dt} = \frac{1}{\cos p}$

Recall $\cos^2 y + \sin^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1 - \sin^2 y}$

Remember $t = \sin p$

$\cos p = \sqrt{1 - t^2}$

Hence, $dp/dt = \frac{1}{\sqrt{(1-t^2)}}$

5) $F(x) = 2x^2 - 5$ ; $g(x) = 4x - 2$

i) $F o g(x) = 2(4x-2)^2 - 5$

$\cancel{= 2(16x^2 +}$

$= 2(4x-2)(4x-2) - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

ii) $g o F(x) = 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

Show that

6) $f_e(x) + f_o(x) = F(x)$.

$F(x) = 3x^2 - 2x + 1$

$f_e(x) = \frac{f(x) + F(-x)}{2}$

$F(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= -4x/2 = -2x$

$F_e(x) + F_o(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$

Sol

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ eqn ①

$[y = \cos x]$

Recall,

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — ②

Comparing ① & ②

$A + B = x + \delta x$ — ③

$A - B = x$ — ④

Adding ③ & ④ & subtracting ③ & ④

$2A = 2x + \delta x$ & $\boxed{B = \delta x/2}$

$A = \frac{2x + \delta x}{2}$

$A = A + \delta x/2$

Comparing ① & ②

$\delta y = \cos \delta x - \cos x$

$= 2\sin(x + \delta x/2)\sin(\delta x/2)$

Dividing through by $\delta x$

$\frac{\delta y}{\delta x} = \frac{-2\sin(x + \frac{\delta x}{2})\sin(\delta x/2)}{\delta x}$

$\frac{\delta y}{\delta x} = \frac{-\sin(x + \delta x/2)\sin(\delta x/2)}{\delta x/2}$

$= -\sin(x + \delta x/2) \times \frac{\sin(\delta x/2)}{\delta x/2}$

Taking limit $\delta x \to 0$

$\lim_{\delta x \to 0} \frac{\sin \delta x/2}{\delta x/2} = 1$

$\delta y/\delta x = -\sin(x + 0/2) \times 1$

$\lim \delta x \to 0$

$\delta y/\delta x = -\sin x$

8) $y = 3t^2$; $x = 1/t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dy}{dt} = 6t$; $\frac{dx}{dt} = \frac{-2}{t^3}$

$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$

$= 6t \times \frac{-2}{t^3} = \frac{6 \times -2}{t^2} = \frac{-12}{t^2}$

$dy/dx = \frac{-12}{t^2}$ //

9) $y = x^2 \cos 2x\, e^{4x}$

Sol

Taking $\log_e$ of both sides

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

Differentiating both with respect to $x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

Multiplying both sides by 'y'

$\frac{dy}{dx} = y\left(2/x - \frac{2\sin 2x}{\cos 2x} + 4\right)$

$= x^2 \cos 2x\, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$\frac{dy}{du} = \cos u$

$\frac{du}{dx} = 9x^2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \cos u \times 9x^2$

$= 9x^2 \cos u$

$= 9x^2 \cos 3x^3 + 5.$