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1) The function is defined for all real numbers except $x = 2$. The set of real numbers except $x = 2$ is the Domain while the real numbers except $y = 0$ is the codomain

2) $K = \ln v$

$$\frac{dK}{dv} = \frac{1}{v}$$

3)a $2x - 3y - 2 = 0 = -2x + 3y + 2 = 0$

$+3y = 2 \cancel{+2x}\ 2x - 2$

$2x - 2 = 3y$

$$y = \frac{2x - 2}{3}$$

b $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

4) $P = \sin^{-1} t$, derivative of P

$P = \frac{t}{\sin}$; $t = \sin P$

$\frac{dt}{dP} = \cos P$; $\frac{dP}{dt} = \frac{1}{\cos P}$

Recall $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm\sqrt{1 - \sin^2 y}$

$\therefore \cos P = \sqrt{1 - t^2}$

Hence $\frac{dP}{dt} = \frac{1}{(1 - t^2)}$

5) $fog(x) = f(g(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$2(4x-2)^2-5$

$2(4x-2)(4x-2)-5$

$2(16x^2-8x-8x+4)-5$

$32x^2-16x-16x+8-5$

$32x^2-32x+3$

(b) $gof(x) = g(f(x))$

$g(x) = 4x-2$

$f(x) = 2x^2-5$

$\therefore 4(2x^2-5)-2$

$8x^2-20-2$

$8x^2-22$

(6) $f(x) = 3x^2-2x+1$. Show that $f_e(x) + f_o(x) = f(x)$

Soln

$f_e(x) = \frac{f(x)+f(-x)}{2}$

$f(-x) = 3(-x)^2-2(-x)+1$

$f(x) = 3x^2+2x+1$

$f_e(x) = \frac{3x^2-\cancel{2x}+1+3x^2+\cancel{2x}+1}{2}$

$\frac{6x^2+2}{2} = \frac{\cancel{2}(3x^2+1)}{\cancel{2}}$

$f_e(x) = 3x^2+1$

$f_o(x) = \frac{3x^2-2x+1-(3x^2+2x+1)}{2}$

$= \frac{\cancel{3x^2} \overset{-2x}{\cancel{-2x}} + \cancel{1} - \cancel{3x^2} \overset{-2x}{\cancel{-2x}} \cancel{-1}}{2}$

$= \frac{-4x}{2} = -2x$

$\therefore f(x) = f_e(x) + f_o(x)$

$= 3x^2+1-2x$

$= 3x^2-2x+1$

7 $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Substract y from both sides

$\delta y = \cos(x + \delta x) - y$

but $y = \cos x$

$\therefore \delta y = \cos(x + \delta x) - \cos x \longrightarrow (*)$

consider from trig

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(A + B) = \cos(A - B) - 2\sin A \sin B$

compare $(*)$ and $2(*)$

Let $A + B = x + \delta x$ --- (i)

$A - B = x$ --- (ii)

adding i and ii

$2A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$\left.\begin{array}{l} A = x + \delta x/2 \\ B = \delta x/2 \end{array}\right\} 3*$

Eqn $(*)$

Compare $(*)$ and $(2*)$

$\cos(x + \delta x) - \cos x = -2\sin(x + \delta x/2)\sin(\delta x/2)$

$\delta y = -2\sin(x + \delta x/2)\sin(\delta x/2)$

$\frac{\delta y}{\delta x} = \frac{-2\sin(x + \delta x/2)\sin(\delta x/2)}{\delta x}$

$\frac{\delta y}{\delta x} = -\sin(x + \delta x/2)\frac{\sin(\delta x/2)}{\delta x/2}$

* A standard limit

$\lim_{\delta x \to 0} \frac{\sin(\delta x/2)}{\delta x/2} = 1$

Find limit of $4(*)$ as $\delta x \to 0$

$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin(x + \delta x/2)\frac{\sin(\delta x/2)}{\delta x/2}$

$= -\sin(x+0)$

$= -\sin x$

$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$

Hence if $y = \cos x$, then $\frac{dy}{dx} = -\sin x$

8 $y = 3t^2$; $x = 1/t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dt} = 6t$; $\frac{dx}{dt} = \frac{-2}{t^3}$

$\frac{dy}{dx} = \frac{6t}{\frac{-2}{t^3}} = 6t \div \frac{-2}{t^3} = \frac{6t \times t^3}{-2}$

$= \frac{6t^4}{-2} = -3t^4$

9 Find $\frac{dy}{dx}$ if $y = x^2 \cos 2x \, e^{4x}$

Taking log of both sides

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

Differentiating both ln + x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

multiply both sides by y

$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$

$\frac{dy}{dx} = x^2 \cos 2x \, e^{4x} \cdot \frac{2\sin x}{\cos x} + 4$

10 $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$\frac{dy}{du} = \cos u$

$\frac{du}{dx} = 9x^2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\cos u \times 9x^2$

$= 9x^2 \cos u$

Recall $u = 3x^3 + 5$

$\therefore 9x^2 \cos u = 9x^2 \cos 3x^3 + 5$