OBIDINMA BLESSING CHINONSO
PHARMACY
19/mths 11/091

(1) x is defined under all real numbers except 2

- Domain - all real numbers except 2
- co-domain - all real numbers except 0

(2) i. $K = \ln V$, differentiate K

$$\frac{dK}{dV} = \frac{1}{V}$$

3a) $2x - 3y - 2 = 0$

$2x - 2 - 3y = 0$

$3y = 2x - 2$

$y = \frac{2x-2}{3}$

b $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

4 $p = \sin^{-1} t$

$p = \frac{t}{\sin}$

$t = \sin p$ — (1)

$\frac{dt}{dp} = \cos p$

$\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin^2 p}$

$\sin p = t$

$\cos p = \sqrt{1 - t^2}$

$\frac{dt}{dp} = \cos p$

$\frac{dp}{dt} = \frac{1}{\cos p}$

$\frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$

5 $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

(a) $fog(x) = 2(4x-2)^2 - 5$

$= 2(4x-2)(4x-2) - 5$

$= 2[16x^2 - 8x - 8x + 4) - 5$

$= 2[16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

(b) $gof(x) = 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

(6) ~~$f(x) = 3x^2 - 2x$~~ $fe(x) = \frac{f(x) + f(-x)}{2}$

$f(x) = 3x^2 - 2x + 1$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$fe(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$

$= \frac{6x^2 + 2}{2}$

$= 3x^2 + 1$

$fo(x) = \frac{f(x) - f(-x)}{2}$

$= \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \frac{-4x}{2}$

$= -2x$

$\therefore fe(x) + fo(x) = f(x)$

7) $y = \cos x$

$y + dy = \cos(x + dx)$

$dy = \cos(x + dx) - y$

$dy = \cos(x + dx) - \cos x$ — (*)

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ — (I)

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ — (II)

(I) − (II)

$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$ — (2*)

Compare (*) and 2*

$(A+B) = (x + dx)$

$(A - B) = x$

$2A = 2x + dx$

$$\left.\begin{aligned} A &= \frac{2x + dx}{2} \\ B &= \frac{dx}{2} \end{aligned}\right\} (3*)$$

Compare (*) and (3*)

$$\cos(x+dx) - \cos x = -2\sin\left[x + \frac{dx}{2}\right]\sin\left[\frac{dx}{2}\right]$$

$$dy = \cos(x+dx) - \cos x$$

$$dy = -2\sin\left[x + \frac{dx}{2}\right]\sin\left[\frac{dx}{2}\right]$$

divide through by dx

$$\frac{dy}{dx} = \frac{-2\sin\left(x + \frac{dx}{2}\right]\sin\left[\frac{dx}{2}\right]}{dx}$$

$$dy/dx = \frac{-\sin\left(x + \frac{dx}{2}\right]\sin\left[\frac{dx}{2}\right]}{dx/2}$$

$$dy/dx = -\sin\left[x + \frac{dx}{2}\right]\frac{\sin\left[\frac{dx}{2}\right]}{dx/2}$$

A standard limit

$$\lim_{dx \to 0} \frac{\sin\left[\frac{dx}{2}\right]}{dx/2} = 1$$

$$\lim_{dx \to 0} \frac{dy}{dx} = \lim_{dx \to 0} -\sin\left(x + \frac{dx}{2}\right]\frac{\sin\left(\frac{dx}{2}\right)}{dx/2}$$

$$= -\sin(x+0)\cdot 1$$

$$= -\sin x$$

$$\lim_{dx \to 0} \frac{dy}{dx} = -\sin x$$

8 $y = 3t^2$ and $x = 1/t^2$

$$dy/dx = \frac{dy/dt}{dx/dt}$$

$$dy/dt = 6t$$

$$dx/dt = -2t^{-3}$$

$$dy/dx = 6t \div -2t^{-3}$$

$$= -3t^3$$

9) $y = x^2 \cos 2x e^{4x}$

find $\log_e$ of both sides

$$\ln y = \ln\left(x^2 \cos 2x e^{4x}\right)$$

$$\ln y = \ln(x^2) + \ln(\cos 2x) + \ln(e^{4x})$$

differentiate wrt x

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\ln(x^2) + \frac{d}{dx}\ln(\cos 2x) + \frac{d}{dx}\ln(e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2} + \frac{-2\sin 2x}{\cos 2x} + \frac{4e^{4x}}{e^{4x}}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2} + \frac{-2\sin 2x}{\cos 2x} + 4$$

$$\frac{dy}{dx} = y\left[\frac{2x}{x^2} - \frac{2\sin 2x}{\cos 2x} + 4\right]$$

$$\frac{dy}{dx} = x^2\cos 2x e^{4x}\left[\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right]$$

10) $y = \sin(3x^3 + 5)$

let $u = 3x^3 + 5$

$$\frac{du}{dx} = 9x^2$$

$$y = \sin u$$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$u = 3x^3 + 5$$

$$\frac{dy}{dx} = 9x^2 \cos 3x^3 + 5.$$