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19/MHS11/150
PHARMACY
MAT 104

1 | Restrict the domain $x \{x: -5 \leq x \leq 53$
The function is defined for all real numbers except $x=2$
Domain - real numbers except $y=0$

2 | $K = \ln y$

$$\frac{du}{dx} = \frac{1}{y}$$

3 | a) $2x - 3y - 2 = 0$

$2x - 2 = 3y$

$$y = \frac{(2x+2)}{3}$$

b) $x^2 + y^2 = 4$

$x^2 - 4 = y^2$

$y = \pm\sqrt{x^2 - 4}$

4 | $p = \frac{t}{\sin}$

$t = \sin p$ --- ①

Recall that; $\sin^2 p + \cos^2 p = 1$ --- ②

$$\frac{dt}{dp} \text{ of } (t) = \cos p$$

from (2) $\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin p}$

$\cos p = \sqrt{1 - t^2}$

$$\frac{dt}{dp} = \cos p = \sqrt{(1-t)^2}$$

$$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$$

5 | $f(x) = 2x^2 - 2$

$g(x) = 4x - 2$

$fog(x) = f(4x - 2)$

$= 2(4x - 2)^2 - 5$

$= 2(4x-2)(4x-2)$

$= 2(16x^2 - 8x - 8x + 4) - 5$

$= 2(32x^2 - 16x - 16x + 8) - 5$

$= 64x^2 - 64x + 3$

b) $g \circ f(x) = g(f(x))$

$g(x) = 4x - 2$

$g(fx) = g(2x^2 - 5)$

$= 4 \cdot (2x^2 - 5) - 2$

$= 8x - 22$

6. $fe(x) = \dfrac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$f(-x) = 3x^2 + 2x + 1$

$fe(x) = \dfrac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$

$= \dfrac{6x^2 + 2}{2}$

$\dfrac{\cancel{2}(3x^2 + 1)}{\cancel{2}}$

$fe(x) = 3x^2 + 1$

$fo(x) = \dfrac{\cancel{3x^2} - 2x + 1 - \cancel{3x^2} + 2x + 1}{2}$

$fo(x) = -\dfrac{\cancel{4}x}{\cancel{2}}$

$= -2x$

$fx = fex + fo(x)$

$= 3x^2 + 1 + (-2x)$

$= 3x^2 - 2x + 1$

$\therefore f(x) = fe(x) + fo(x)$

from first principle

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Subsract y from both sides

$\delta y = \cos(x + \delta x) - y$

$\delta y = \cos(x + \delta y) - \cos x \quad --- (1)$

Consider from trig

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B \quad --- (2)$

compare (1) & (2)

let $A+B = x + \delta x \quad --- (i)$

$A+B = x \quad --- (ii)$

Adding (i) & (ii)

$2A = 2x + \delta x$

$A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$$\left.\begin{aligned} A &= x + \frac{\delta x}{2} \\ B &= \frac{\delta x}{2} \end{aligned}\right\} - (3)$$

compare (1) & (2)

$\cos(x + \delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\therefore \delta y = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\frac{\delta y}{\delta x} = \frac{2\sin\left(x + \frac{\delta x}{2}\right)\sin(\delta x/2)}{\delta x}$

$\frac{\delta y}{\delta x} = \frac{-\sin\left(x + \frac{\delta x}{2}\right)\sin(\delta x/2)}{\delta x/2}$

$\frac{\delta x}{\delta x} = -\sin\left(x + \frac{\delta x}{2}\right)\frac{\sin(\delta x/2)}{\delta x/2} \quad --- (4)$

standard limit

$\lim_{\delta x \to 0} \frac{\sin(\delta x/2)}{\delta x/2} = 1$

find limit of $\frac{\delta y}{\delta x}$ as $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} . \sin\left(x + \frac{\delta x}{2}\right) \sin \frac{(\delta x/2)}{\delta x/2}$$

$$= -\sin(x + 0) \cdot 1$$

$$= -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

8 $y = 3t^2 \quad x = 1/t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = 3t^2 = 6t$$

$$\frac{dx}{dt} = t^{-2}$$

$$= -2t^{-3}$$

$$= \frac{-2}{t^3}$$

$$\frac{dy}{dx} = \frac{6t}{-2t^{-3}}$$

$$= 6t \times \frac{1}{\frac{-2}{t^3}}$$

$$= 6t \times \frac{t^3}{-2}$$

$$= -3t^4$$

9 $\ln y = \ln(x^2 \cos 2x \, e^{4x})$

$$\ln y = \ln(x^2) + \ln(\cos 2x) + \ln(e^{4x})$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x^2} - 2\tan 2x - 4$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

$$= x^2 \cos 2x \, e^{4x}\left(\frac{2}{x} - 2t2x + 4\right)$$

b) using chain rule:

$y = \sin(3x^3+5)$

let $u = 3x^3+5$ $\quad \frac{du}{dx} = 9x^2$

$y = \sin u$ $\quad \frac{dy}{du} = \cos u$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 9x^2$

$\frac{dy}{dx} = 9x^2 \cos u$

but $u = 3x^3+5$

$\frac{dy}{dx} = 9x^2 \cos(3x^3+5).$