ELUE FLOURISH OKOLIE
19/MHS 11/052
PHARMACY

1 For what value of x is the function $y = \frac{1}{x-2}$ defined
State the domain and codomain

solution

The expression is undefined when the denominator is equal to zero

$x - 2 = 0$

$x = 2$

The Domain is all integers of x except for $x = 2$.

i.e

$x < 2$, $x > 2$,

$x \neq 2$

The codomain

$x < 2$

The value $1, -1, -2$, are less than 2

so when $x = 1$

$y = \frac{1}{x-2} = \frac{1}{1-2} = \frac{1}{-1}$

$y = -1$

when $x = -1$

$y = \frac{1}{-1-2} = \frac{1}{-3}$

when $x = -2$

$y = \frac{1}{-2-2} = \frac{-1}{4}$

For $x > 2$ it implies that $x = 3, 4 \ldots$

when $x = 3$

$y = \frac{1}{3-2} = \frac{1}{1} = 1$

when $x = 4$

$y = \frac{1}{4-2} = \frac{1}{2}$

Q2:

If $K = \ln V$ differentiate K

let $K = \ln V$

$\frac{dK}{dV} = \frac{1}{V}$

Q3: Express y and an explicit function of $\ln x$ in the following

a) $2x - 3y - 2 = 0$

b) $x^2 + y^2 = 4$

Solution

Explicit function

$2x - 3y - 2 = 0$

$2x - 2 = 3y$

$\frac{2(x-1)}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$y = \frac{2}{3}(x-1)$

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

OR

$y = -\sqrt{4-x^2}$

ALSO

Implicit function

$x^2+y^2=4$

$2x + 2y\, dy/dx = 0$

$\frac{2y\, dy/dx}{2y} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

Q4

If $P = \sin^{-1} t$, find the derivative of P

$P = \sin^{-1} t$

$t = \sin p$

$\frac{dt}{dp} = \cos p$

$\frac{dp}{dt} = \frac{1}{\cos p}$

$\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos^2 p = 1 - (\sin p)^2$

$\cos p = \sqrt{1-t^2}$

$\cos p = \sqrt{1-t^2}$

$\frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$

Q5

If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$

find $f \circ g(x)$ and $g \circ f(x)$

solution

$f \circ g(x)$

$f(4x-2)$

$2(4x-2)^2 - 5$

$2[(4x-2)(4x-2)] - 5$

$2[16x^2 - 8x - 8x + 4] - 5$

$32x^2 - 32x + 8 - 5$

$32x^2 - 32x + 3$

$G \circ f(x)$

$G(2x^2-5)$

$4(2x^2-5) - 2$

$8x^2 - 20 - 2$

$8x^2 - 22$

Q7 (First Principle)

$y = \cos x$

Increase y by Δy and x by Δx

$y + \Delta y = \cos(x + \Delta x)$

$\Delta y = \cos(x + \Delta x) - y$

$\Delta y = \cos(x+\Delta x) - \cos x$

Δy Let $Q = x + \Delta x$, $P = x$

$\cos Q - \cos P$

$\Delta y = 2\sin\left(\frac{x + x + \Delta x}{2}\right) \sin\left(\frac{x - (x+\Delta x)}{2}\right)$

$\Delta y = 2\sin\left(\frac{2x+\Delta x}{2}\right) \sin\left(\frac{\cancel{x} - \cancel{x} - \Delta x}{2}\right)$

$\Delta y = 2\sin\left(\frac{2x}{2} + \frac{\Delta x}{2}\right) \sin\left(-\frac{\Delta x}{2}\right)$

$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(x + \frac{\Delta x}{2}\right) \sin\frac{\Delta x}{2}}{\Delta x}$

$$\frac{\Delta y}{\Delta x} = -\frac{2\sin}{2}\left(x+\frac{\Delta x}{2}\right)\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\frac{\sin\frac{\Delta x}{2}}{\Delta x/2}$$

As $\Delta x \to 0$

$$\frac{\sin\left(\Delta x/2\right)}{\Delta x/2} \to 1$$

$$\frac{\Delta y}{\Delta x} = -\sin\left(x+\frac{\Delta x}{2}\right)$$

As $\Delta x \to 0$

$$\frac{\Delta y}{\Delta x} = -\sin\left(x+0/2\right)$$

$$\frac{\Delta y}{\Delta x} = -\sin(x+0)$$

limit $\frac{\Delta y}{\Delta x} = -\sin x$

$\Delta x \to 0$

Hence $dy/dx = -\sin x$

Question 8

Find dy/dx of $y = 3t^2$ and $x = 1/t^2$

Solution

$y = 3t^2$

but $x = 1/t^2$

$1/x = t^2/1$

$t^2 = 1/x$

$y = 3 \times (1/x)$

$y = 3x^{-1}$

$dy/dx = -3x^{-1-1}$

$= -3x^{-2}$

Q9

$y = x^2 \cos 2x \; e^{4x}$

Solution

taking $\log_e$ of both sides

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

differentiating both wrt x

$$1/y \; dy/dx = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

multiplying both sides by 'y'

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

$$= x^2\cos 2x\, e^{4x} * \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$$

Q10

Given that

$y = \sin(3x^3+5)$ find the derivative of y

Solution

$y = \sin(3x^3+5)$

let $u = 3x^3+5$

$$\frac{du}{dx} = 9x^2$$

$y = \sin u$

$$\frac{dy}{du} = \cos u \Rightarrow \cos(3x^3+5)$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$\frac{dy}{dx} = 9x^2 \times \cos u$$

$$\frac{dy}{dx} = 9x^2\cos(3x^3+5)$$