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DEPARTMENT: COMPUTER SCIENCE

MATRIC NO: 19/SC1011015

ASSIGNMENT

Integrate the following with respect to their variable

$$① \int \frac{11-3x}{x^2+2x-3}$$

Solution

$$\int \frac{11-3x}{x^2+2x-3} = \int \frac{11-3x}{(x-1)(x+3)}$$

Resolve

$$11-3x = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

Find the L.C.M of the right hand side

$$11-3x = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$11-3x = A(x+3) + B(x-1)$$

$$\text{Let } x = -3$$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$20 = A(0) - 4B$$

$$\frac{20}{-4} = \frac{-4B}{-4}$$

$$-5 = B$$

$$B = -5$$

$$\text{Let } x = 1$$

$$11-3(1) = A(1+3) + B(1-1)$$

$$8 = 4A + B(0)$$

$$\frac{8}{4} = \frac{4A}{4}$$

$$2 = A$$

$$A = 2$$

$$\therefore \text{Resolve} = \frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$\Rightarrow \int \frac{11-3x}{x^2+2x-3} = \int \frac{2}{(x-1)} dx - \int \frac{5}{(x+3)} dx$$

$$= 2 \int \frac{1}{(x-1)} - 5 \int \frac{1}{(x+3)}$$

$$= 2 \ln |(x-1)| - 5 \ln |(x+3)| + C$$

$$(2) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

Solution

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

Find the L.C.M of the R.H.S

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equate the numerator of the L.H.S to the numerator of the R.H.S

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$\text{Let } x = -3$$

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$$

$$2(-3)^2 - 9(-3) - 35 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$2(-3)^2 - 9(-3) - 35 = 0 + 0 + 10C$$

$$\frac{10}{10} = \frac{10C}{10}$$

$$C = 1$$

$$\text{Let } x = -1$$

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$-24 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$-24 = -6A + 0 + 0$$

$$\frac{-24}{-6} = \frac{-6A}{-6}$$

$$A = 4$$

$$\text{Let } x = 2$$

$$2(2)^2 - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$-45 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$-45 = 0 + 15B + 0$$

$$-45 = 15B$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$B = -3$$

$$\therefore \text{Resolve} = \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}$$

$$\Rightarrow \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \int \frac{1}{x+1} - 3 \int \frac{1}{x-2} + \int \frac{1}{x+3}$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$

$$\textcircled{3} \int \frac{1}{x^2+121} dx$$

Solution

$$\int \frac{1}{x^2+121} dx = \int \frac{1}{121(x^2+1)} = \frac{1}{121} \int \frac{1}{x^2+1} = \frac{1}{121} \int \frac{1}{\left(\frac{x}{11}\right)^2+1} dx$$

$$u = \frac{x}{11}$$

$$\frac{du}{dx} = \frac{1}{11}$$

$$dx = 11 du$$

$$= \frac{1}{121} \int \frac{1}{u^2+1} dx = \frac{1}{121} \int \frac{1}{u^2+1} \cdot 11 du = \frac{1}{11} \int \frac{1}{u^2+1} du$$

Recall, $\int \frac{1}{u^2+1} du$ is a standard integral, therefore = $\arctan(u)$

$$= \frac{1}{11} \arctan u + C$$

Recall, $u = x$

$$= \frac{1}{11} \arctan\left(\frac{x}{11}\right) + C$$

$$= \arctan\left(\frac{x}{11}\right) + C$$

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