

$$= -\frac{4x}{2} = -2x$$

$$\therefore f(x) = f(x+h) + f(x-h)$$

$$f(x) = 3x^2 + 1 - 2x$$

$$= \underline{3x^2 - 2x + 1}$$

7) Differentiate $y = \cos x$ from first principle
solution

$$y = \cos x$$

$$y + \delta y = \cos(x + \delta x)$$

Subtract y from both sides

$$\delta y = \cos(x + \delta x) - y$$

$$y = \cos x$$

$$\delta y = \cos(x + \delta x) - \cos x \quad \text{--- } (*)$$

consider from trig

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) - \cos(A - B)$$

$$= -2 \sin A \sin B \quad \text{--- } (2*)$$

compare equ $*$ and $2*$

let

$$A + B = x + \delta x \quad \text{--- } (1)$$

$$A - B = x \quad \text{--- } (1)$$

Adding (1) and (1)

$$2A = 2x + \delta x$$

$$A = \frac{2x + \delta x}{2}$$

$$A = x + \frac{\delta x}{2} \quad \text{--- } 3*$$

$$B = \frac{\delta x}{2}$$

compare $*$ and $2*$

$$\cos(x + \delta x) - \cos x = -2 \sin\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right)$$

D) $(g \circ f)(x)$

$g(f(x))$

$$f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$g(2x^2 - 5)$$

$$4(2x^2 - 5) - 2$$

$$8x^2 - 20 - 2$$

$$8x^2 - 22$$

Q) If $F(x) = 3x^2 - 2x + 1 = 0$. Show that $F_e(x) + F_o(x) = F(x)$

Solution

$$F_e(x) = \frac{F(x) + F(-x)}{2}$$

$$= \frac{F(x) + F(-x)}{2} = \frac{3(x)^2 - 2(x) + 1 + 3(-x)^2 - 2(-x) + 1}{2}$$

$$= \frac{3x^2 + 2x + 1}{2}$$

$$= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{3x^2 + 3x^2 - 2x + 2x + 2}{2}$$

$$= \frac{6x^2 + 2}{2}$$

$$= \frac{2(3x^2 + 1)}{2}$$

$$= 3x^2 + 1$$

$$F_o = \frac{F(x) - F(-x)}{2}$$

$$= \frac{(3x^2 - 2x + 1) - (3x^2 + 2x + 1)}{2}$$

$$= \frac{3x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$= \frac{3x^2 - 3x^2 - 2x - 2x - 1 + 1}{2}$$

$$= \frac{3x^2 - 3x^2 - 2x - 2x - 1 + 1}{2}$$

$$4) p = \frac{t}{\sin}$$

$$t = \sin p \quad \dots (1)$$

$$\text{Recall that, } \sin^2 p + \cos^2 p = 1 \quad \dots (2)$$

$$\frac{dt}{dp} \text{ of (1)} = \cos p$$

$$\text{From equ 2} = \sin^2 p + \cos^2 p = 1$$

$$\cos^2 p = 1 - \sin^2 p$$

$$\cos p = \sqrt{1 - \sin^2 p}$$

$$\cos p = \sqrt{1 - t^2}$$

$$\frac{dt}{dp} = \cos p = \sqrt{1 - t^2}$$

$$\frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$$

5) If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find
 $f \circ g(x)$ and $g \circ f(x)$

$$\text{If } f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

a) $f \circ g(x)$

b) $g \circ f(x)$

Solution

a) $(f \circ g)(x)$

$$f(g(x))$$

$$f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$(f \circ g)(x) = f(g(x))$$

$$f(4x - 2)$$

$$2(4x - 2)^2 - 5$$

$$2(4x - 2)(4x - 2) - 5$$

$$2(16x^2 + 4 - 8x - 8x) - 5$$

$$2(16x^2 - 16x + 4) - 5$$

$$32x^2 - 32x + 8 - 5$$

$$\underline{\underline{32x^2 - 32x + 3}}$$

$$\delta y = -2 \sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})$$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})}{\frac{\delta x}{2}}$$

To find a standard limit

$$\lim_{\delta x \rightarrow 0} \frac{\sin(\frac{\delta x}{2})}{\frac{\delta x}{2}} = 1$$

$$\lim_{\delta x \rightarrow 0} = \frac{-\sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})}{\frac{\delta x}{2}}$$

$$= -\sin(x + 0) \cdot 1$$

$$= -\sin x$$

$$\therefore \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

$$\therefore \frac{dy}{dx} = \underline{\underline{-\sin x}}$$

8) Find $\frac{dy}{dx}$ if $y = 3t^2$ and $x = \frac{1}{t^2}$

$$\text{If } y = 3t^2$$

$$x = \frac{1}{t^2}$$

find $\frac{dy}{dx}$

$$y = 3t^2, \quad \frac{dy}{dt} = 6t$$

$$x = \frac{1}{t^2}, \quad \frac{dx}{dt} = -2t^{-3} = \frac{-2}{t^3}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{6t}{-2} = \frac{6t \times t^3}{-2}$$

$$\frac{dx}{dt} = \frac{-2}{t^3} = -2t^{-3}$$

9) Find $\frac{dy}{dx}$, if $y = x^2 \cos^2 2x e^{4x}$

Solution

$$\ln y = \ln (x^2 \cos^2 2x e^{4x})$$

$$\ln y = \ln x^2 + \ln \cos^2 2x + \ln e^{4x}$$

$$\frac{d}{dx} (\ln y) = \frac{d}{dx} (\ln x^2) + \frac{d}{dx} (\ln \cos^2 2x) + \frac{d}{dx} (\ln e^{4x})$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x) + \frac{1}{e^{4x}} (4e^{4x})$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - 2 \frac{\sin 2x}{\cos 2x} + 4$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - 2 \tan 2x + 4$$

Multiplying both sides by y , we have

$$\frac{dy}{dx} = y \left(\frac{2}{x} - 2 \tan 2x + 4 \right)$$

$$\text{but } y = x^2 \cos^2 2x e^{4x}$$

$$\frac{dy}{dx} = x^2 \cos^2 2x e^{4x} \left(\frac{2}{x} - 2 \tan 2x + 4 \right)$$

10) $y = \sin(3x^3 + 5)$

$$\text{let } u = 3x^3 + 5$$

$$y = \sin u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{du} = \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$\text{but } u = 3x^3 + 5$$

$$\frac{dy}{dx} = \underline{9x^2 \cos(3x^3 + 5)}$$

DATE :- Wednesday 8th April, 2020.

COURSE :- MATHS 104 ASSIGNMENT

1) For what values of x is the function $y = \frac{1}{x-2}$ defined?
state the domain and co-domain

Solution

IS not defined because is a function, because of the denominator.

The function is defined for all real numbers except $x = 2$

Domain = Real numbers except $x = 2$

Codomain = Real numbers except $y = 0$

2) IF $k = \ln v$: differentiate k

$$\frac{dk}{dv} = \frac{1}{v}$$

3) Express y as an explicit function of x in the following

a) $2x - 3y - 2 = 0$

b) $x^2 + y^2 = 4$

Solution

a) $2x - 3y - 2 = 0$

$$2x - 3y = 2$$

$$-\frac{3y}{-3} = \frac{2-2x}{-3}$$

$$y = \frac{-2+2x}{3}$$

b) $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \sqrt{4-x^2}$$

$$y = \pm \sqrt{4-x^2}$$