

Adeyige Samuel Ademideun,
1912011004,
Computer Science,
MAT 102.

1) $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$

Velocity = $\frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k$

$= (14t)i + (12t - 4)j + (1)k$

Velocity = $\frac{dr}{dt}$

$\frac{dr}{dt} = 14ti + (12t - 4)j + k$

2) $A = 2i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$. Find $A \times (B \times C)$.

$B \times C =$

$$\begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix} = i \begin{vmatrix} 4 & -3 \\ 0 & -3 \end{vmatrix} - j \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$= i [12 - 0] - j [-6 - 0] + k [8 - 0]$

$= 12i + 6j + 8k$

$A \times (B \times C) =$

$$\begin{vmatrix} i & j & k \\ 2 & 2 & -4 \\ 12 & 6 & 8 \end{vmatrix} = i \begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 2 & -4 \\ 12 & 8 \end{vmatrix} + k \begin{vmatrix} 2 & 2 \\ 12 & 6 \end{vmatrix}$$

$= i [16 - 24] - j [16 - 48] + k [12 - 24]$

$= -8i + 32j - 12k$

$= 40i + 28j - 4k$

$$\begin{aligned}
 \text{4. (c) } \vec{R} &= 4s \cos 3t \vec{i} + 4e^{3t} + 7t^3 \vec{k}, \\
 \int \vec{R} &= \int 4s \cos 3t \vec{i} + \int 4e^{3t} + \int 7t^3 \vec{k}, \\
 \int \vec{R} &= 4i \int s \cos 3t + 4j \int e^{3t} + 7k \int t^3, \\
 \int \vec{R} &= \frac{4i}{3} (-\cos 3t) + \frac{4j}{3} e^{3t} + \frac{7}{4} k t^4.
 \end{aligned}$$

$$\int \vec{R} = -\frac{4}{3} \cos(3t) \vec{i} + \frac{4e^{3t}}{3} \vec{j} + \frac{7t^4}{4} \vec{k},$$

$$\begin{aligned}
 \text{4. (d) } \vec{A} &= 7\vec{i} + 2\vec{j} - \vec{k}, \quad \vec{B} = 2\vec{i} + \vec{j} + 4\vec{k}, \quad \vec{C} = \vec{i} + \vec{j} + \vec{k}, \quad \text{find } (\vec{A} + \vec{C}) \cdot (\vec{B} - \vec{A}), \\
 (\vec{A} + \vec{C}) \cdot (\vec{B} - \vec{A}) &= [(7\vec{i} + 2\vec{j} - \vec{k}) + (\vec{i} + \vec{j} + \vec{k})] \cdot [(2\vec{i} + \vec{j} + 4\vec{k}) - (7\vec{i} + 2\vec{j} - \vec{k})] \\
 &= [8\vec{i} + 3\vec{j} + 0\vec{k}] \cdot [-5\vec{i} - \vec{j} + 5\vec{k}] \\
 &= -40 + 0 - 5 \\
 &= -45
 \end{aligned}$$

5) $x = t, y = t^2, z = t^3$, find a unit vector tangent at point where $t = 1$.

$$\vec{T} = \frac{d\vec{r}}{dt} / \left| \frac{d\vec{r}}{dt} \right|$$

$$\begin{aligned}
 \vec{r} &= x\vec{i} + y\vec{j} + z\vec{k} \\
 \vec{r} &= t\vec{i} + t^2\vec{j} + t^3\vec{k}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d\vec{r}}{dt} &= \vec{i} + 2t\vec{j} + 3t^2\vec{k} \quad \text{at } t=1, \frac{d\vec{r}}{dt} = \vec{i} + 2\vec{j} + 3\vec{k}, \\
 &= \vec{i} + 2(\vec{i})\vec{j} + 3(\vec{i})^2\vec{k} \\
 &= \vec{i} + 2\vec{j} + 3\vec{k}
 \end{aligned}$$

$$\begin{aligned}
 \left| \frac{d\vec{r}}{dt} \right| &= \sqrt{(\vec{i})^2 + (2\vec{j})^2 + (3\vec{k})^2} \\
 &= \sqrt{1^2 + 2^2 + 3^2} \\
 &= \sqrt{14 + 9} \\
 &= \sqrt{23} = 3.74
 \end{aligned}$$

$$\text{Hence, } \vec{T} = \frac{\vec{i} + 2\vec{j} + 3\vec{k}}{3.74}$$