Dept: Computer Engineering

Matric No: 18/ENG02/042 [CO]

Course: Mat 104

Differentiate the following

1. $y = [(x+1)^2 (x-2)^{1/2}] / [(2x-1)(x-3)^{4/3}]$
2. $y = [3e^k \sin 2k] / k^{5/2}$

Integrate the following with respect to the variable

1. $4\sec^2(3m+1)$
2. $2t(3t^2-1)^{1/2}$
3. $2x/(4x^2-1)^{1/2}$

Solution

1) $y = \dfrac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x+3)^{3/2}}$

$\ln y = \ln((x+1)^2) + \ln(\sqrt{x-2}) - \ln(2x-1) - \ln((x+3)^{3/2})$

$(1/y)\dfrac{dy}{dx} = \dfrac{1}{(x+1)^2} \cdot 2(x+1) + \dfrac{1}{\sqrt{x-2}} \cdot \frac{1}{2}(x-2)^{-1/2} - \dfrac{1}{2x-1} \cdot 2$

$- \dfrac{1}{(x+3)^{3/2}} \cdot \dfrac{3}{2}(x+3)^{1/2}$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{2}{(x+1)} + \dfrac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \dfrac{2}{2x-1} - \dfrac{3}{2}(x+3)^{1/2-3/2}$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{x+1} + \dfrac{1}{2(x-2)} - \dfrac{2}{2x-1} - \dfrac{3}{2(x+3)}\right]$

$\dfrac{dy}{dx} = \dfrac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x+3)^{3/2}}\left[\dfrac{2}{x+1} + \dfrac{1}{2x-4} - \dfrac{2}{2x-1} - \dfrac{3}{2x+6}\right]$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^x}\cdot 3e^x + \frac{1}{\sin 2x}\cdot 2\cos 2x - \frac{1}{x^{5/2}}\cdot\frac{5}{2}x^{3/2}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2}x^{3/2-5/2}$$

$$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5x^{-1}}{2}\right]$$

$$\frac{dy}{dx} = \frac{3e^x\sin 2x}{x^{5/2}}\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2x}\right]$$

Part 2

(1) $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,du$

$u = 3m+1$

$\frac{du}{dm} = 3$

$du = 3dm$

$dm = \frac{du}{3}$

$4\int \sec^2(u)\,\frac{du}{3}$

$4/3\int \sec^2(u)\,du$

$\frac{4}{3}\tan u + C$

$= \frac{4}{3}\tan(3m+1) + C$

2) $\int 2t(3t^2-1)^{1/2}\,dt$

$u = \sqrt{3t^2-1}$

$u^2 = 3t^2-1$

$3t^2 = u^2+1$

$t^2 = \dfrac{u^2+1}{3}$

$t = \sqrt{\dfrac{u^2+1}{3}}$

$\dfrac{dt}{du} = \dfrac{1}{2}\left(\dfrac{u^2+1}{3}\right)^{-1/2} \cdot \dfrac{2u}{3}$

$\dfrac{dt}{du} = \dfrac{u}{3}\left(\dfrac{u^2+1}{3}\right)^{-1/2}$

$dt = \dfrac{u\,du}{3}\left(\dfrac{u^2+1}{3}\right)^{-1/2}$

$\int 2\left(\dfrac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \dfrac{u\,du}{3}\left(\dfrac{u^2+1}{3}\right)^{-1/2}$

$= \dfrac{2}{3}\int u^2\left(\dfrac{u^2+1}{3}\right)^{1/2-1/2} du$

$= \dfrac{2}{3}\int u^2\,du$

$= \dfrac{2}{3}\left[\dfrac{u^3}{3}\right] + C$

$= \dfrac{2u^3}{9} + C$

$= \dfrac{2(3t^2-1)^{3/2}}{9} + C$ //

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx$$

$$u = \sqrt{4x^2-1}$$

$$u^2 = 4x^2-1$$

$$4x^2 = u^2+1$$

$$x^2 = \frac{u^2+1}{4}$$

$$x = \sqrt{\frac{u^2+1}{4}}$$

$$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$$

$$\frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\int \frac{\cancel{2}^{1}\left(\frac{u^2+1}{4}\right)^{1/2}}{\cancel{u}} \cdot \frac{\cancel{u}\,du}{\cancel{4}_{2}}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\frac{1\cancel{x}}{2}\int\left(\frac{u^2+1}{4}\right)^{1/2-1/2} du$$

$$\frac{1}{2}\int du$$

$$= \frac{u}{2} + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$