

1 $y = \frac{1}{3x} - 2$

- The function is defined for all real numbers except $x=2$

- The domain is the set of real numbers except $x=2$

- The codomain of the set of real number

$y=0$

2 $k = \ln u$

$\frac{dk}{du} = \frac{1}{u}$

3a $2x - 3y - 2 = 0$

$-3y = 2 - 2x$

$y = \frac{2 - 2x}{-3}$

$y = \frac{2x + 2}{3}$

b $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm \sqrt{4 - x^2}$

4. Find $\frac{dp}{dt}$; $p = \sin^{-1} t$

$p = t$; $t = \sin p$

$\frac{dp}{dt} = \cos p$; $\frac{dp}{dt} = \frac{1}{\cos p}$

Recall, $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm \sqrt{1 + \sin^2 y}$

$t = \sin p$

$\therefore \cos p = \sqrt{1 + t^2}$

5 $F(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$F \circ g(x) = 2(4x - 2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$g \circ f(x) = 4(2x^2 - 5) - 2$

6) Show that $f(x) = f_2(x) + f_0(x)$

$f_2(x) = f(x)3x^2 - 2x + 1$

$f_0(x) = 1x + f(-x)$

$f(-x) = 3(-x)^2 - 2(-x) + 1$
 $= 3x^2 + 2x + 1$

$f_2(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \frac{-4x}{2} = -2x$

$f_2(x) + f_0(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ (y = cos x)

Recall $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ (2)

Comparing (1) & (2)

$A+B = x + \delta x$ (3)

$A-B = x$ (4)

Adding (3) & (4) and sub. (3) & (4)

$2A = 2x + \delta x$ & $B = \frac{\delta x}{2}$

$A = \frac{2x + \delta x}{2}$

$A = x + \frac{\delta x}{2}$

$A = x + \frac{\delta x}{2}$

$A = x + \frac{\delta x}{2}$

Comparing 1 & 2

$$S_y = \cos - \cos x$$

$$= 2 \sin(x + \frac{\Delta x}{2}) \sin(\frac{\Delta x}{2})$$

D. b. S by Δx

$$\frac{\Delta y}{\Delta x} = 2 \sin(x + \frac{\Delta x}{2}) \sin(\frac{\Delta x}{2})$$

$$\frac{\Delta y}{\Delta x} = \frac{2 \sin(x + \frac{\Delta x}{2}) \sin(\frac{\Delta x}{2})}{(\Delta x/2)}$$

Taking limit $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x/2}{\Delta x/2} = 1$$

$$\frac{\Delta y}{\Delta x} = \sin(x + 0/2) \times 1$$

$$\frac{\Delta y}{\Delta x} = \sin x$$

8 $y = 3t^2, x = t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = 6t; \frac{dx}{dt} = 2t/3$$

$$\frac{dy}{dx} = 6t \div 2t/3$$

$$= \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

$$\frac{dy}{dx} = \frac{-12}{4^2}$$

7) $y = x^2 \cos 2x e^{+x}$

Taking logs of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{+x}$$

Differentiating both by x

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$$

multiply both sides by y

$$\frac{dy}{dx} = y \left(\frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$= x^2 \cos 2x e^{+x} \times \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$\frac{dy}{dx} = \cos u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos 3x^3 + 5$$