

A) Question 1

Explain the concept of linear programming and its application to engineering.

Linear programming (LP, also called linear optimization) is a method that is used to get the best outcome in a model. This technique is used for a system of linear constraints and a linear objective function, the goal of LP is to find the values of the variables that maximize or minimize the objective function. Linear programming applies matrix algebra to solve a broad class of problems.

The applications of linear programming in engineering include

- i) Cost efficiency
- ii) Shape optimization

5. cost efficiency

$$1000 \geq 1000$$

$$200 \geq 200$$

$$300 \geq 300$$

$$0 \leq 1000$$

6. shape optimization

$$1000 \geq 1000$$

$$200 \geq 200$$

$$\left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 500 \\ 0 & 1 & 2 & 0 & 600 \\ 0 & -5 & 15 & 0 & 1500 \end{array} \right]$$

$$R_1 = -R_2 + R_1$$

$$R_3 = 5R_2 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 200 \\ 0 & 1 & 2 & 0 & 600 \\ 0 & 0 & 0 & 0 & 18000 \end{array} \right]$$

Therefore

$$x_1 = 200$$

$$x_2 = 600$$

$$x_3 = 18,000$$

Now we have to find the value of x_1, x_2, x_3 which are the values of the variables. $x_1 = 200, x_2 = 600, x_3 = 18,000$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & 600 \\ 0 & 0 & 1 & 0 & 18000 \end{array} \right]$$

$$R_1 = 300 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 300 \\ 0 & 1 & 0 & 0 & 600 \\ 0 & 0 & 1 & 0 & 18000 \end{array} \right]$$

Therefore the solution is $x_1 = 300, x_2 = 600, x_3 = 18000$

Converting into matrix form

$$\begin{array}{c|ccc|c}
 & y & S_1 & S_2 & Z \\
 \hline
 (1) & 1 & 1 & 0 & 1000 \\
 (2) & 1 & 0 & 1 & 800 \\
 \hline
 & -20 & 0 & 0 & 0 \\
 \hline
 & & & & 1000/2 = 500 \\
 & & & & 800/1 = 800
 \end{array}$$

We take the number that is the most negative and pick our pivot element from that column.

We want 2 to become 1 to simplify, therefore we divide the whole row by 2

$$R_1 \div 2 \rightarrow \begin{array}{c|ccc|c}
 (1) & 1/2 & 1/2 & 0 & 500 \\
 (2) & 1 & 0 & 1 & 800 \\
 \hline
 & -20 & 0 & 0 & 0 \\
 \hline
 \end{array}$$

Now we need to make succeeding values of 1 on the same column zero. \therefore we start with 1 (second row second column)

We will apply $-R_1 + R_2 \rightarrow R_2$

$$\begin{array}{c|ccc|c}
 1 & 1/2 & 1/2 & 0 & 500 \\
 0 & 1/2 & -1/2 & 1 & 300 \\
 \hline
 -20 & 0 & 0 & 0 & 0 \\
 \hline
 \end{array}$$

We will now apply $R_3 = 30R_1 + R_3$

$$\begin{array}{c|ccc|c}
 1 & 1/2 & 1/2 & 0 & 500 \\
 0 & 1/2 & -1/2 & 1 & 300 \\
 0 & -5 & 15 & 0 & 15000 \\
 \hline
 \end{array}$$

new pivot multiply row 2 by 2

(3)

Converting into matrix form

$$\begin{array}{c}
 \begin{array}{c} \text{Pivot} \\ \text{element} \end{array} \\
 \left[\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 1 & 1 & 0 & 1000 \\
 1 & 0 & 1 & 800 \\
 -30 & 0 & 0 & 0
 \end{array} \right]
 \end{array}$$

$1000/2 = 500$
 $800/1 = 800$

We take the number that is the most negative and pick our pivot element from that column.

We want 2 to become 1 to simplify, therefore we divide the whole row by 2

$$R_1 \div 2$$

$$\left[\begin{array}{ccc|c}
 (1) & 1/2 & 0 & 500 \\
 \rightarrow 1 & 1 & 0 & 800 \\
 -30 & 0 & 0 & 0
 \end{array} \right]$$

Now we need to make succeeding values of 1 on the same column zero. \therefore we start with 1 (second row second column)

We will apply $-R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c}
 1 & 1/2 & 0 & 500 \\
 0 & 1/2 & -1/2 & 300 \\
 -30 & 0 & 0 & 0
 \end{array} \right]$$

We will now apply $R_3 = 30R_1 + R_3$

$$\left[\begin{array}{ccc|c}
 1 & 1/2 & 0 & 500 \\
 0 & 1/2 & -1/2 & 300 \\
 0 & -5 & 15 & 15000
 \end{array} \right]$$

new pivot

multiply row 2 by 2

2 Question 2

A & B company manufactures printers and keyboards. The contribution margins of the printer and keyboard are 30 naira and 20 naira respectively. Two types of skilled labour are required to manufacture these products: soldering and assembling. A printer requires 2 hours of soldering and 1 hour of assembling. A keyboard requires 1 hour of soldering and 1 hour of assembling. A & B company has 1000 soldering hours and 800 assembling hours available per week. There are no constraints on the supply of raw materials. Demand for keyboards is unlimited, but at most 350 printers are sold each week. Determine the best fit values in order to minimize its weekly total contribution margin.

Solution

Given

x : printers produced per week
 y : keyboards produced per week

Defining objective function, Z

$$Z = 30x + 20y$$

subject to

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x \leq 350$$

$$\text{for } x, y \geq 0$$

Adding slack values to the constraints

$$2x + y + S_1 \leq 1000$$

$$x + y + S_2 \leq 800$$