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19/MHS01/050

MAT 104 Assignment.

1) Examine whether or not these pair of lines are perpendicular to each other.

a) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

b) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

2) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1$ and $y = 2$

Solution.

a) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

A line is said to be perpendicular if $m_1 m_2 = -1$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$m_1 = \frac{y + 1}{x + 3} = \frac{1}{3}$$

$$m_2 = \frac{y - 3}{x - 1} = \frac{-3}{+1} = -3$$

$$m_1 m_2 \Rightarrow \frac{1}{3} \times -3 = -1$$

$\therefore y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are not perpendicular to each other.

b) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

$$3y - 4 - 2x - 3 = 0 \text{ and } y - 5 - x - 6 = 0$$

$$3y - 2x - 7 = 0 \text{ and } y - x - 11 = 0$$

$$m_1 = \frac{4-3}{2+2} = \frac{-3}{2}$$

$$m_2 = \frac{4-1}{2-1} = \frac{-1}{-1} = 1$$

$$m_1 m_2 \Rightarrow \frac{-3}{2} \times 1 = \frac{-3}{2} = -1.5$$

$\therefore 3y+4=2x+3$ and $y-5=x+6$ is perpendicular to each other.

9) $x^2 + y^2 + 3xy - 11 = 0$ at $x=1$ and $y=2$

$\frac{dy}{dx}$ = Equation of tangent = $y - y_1 = m(x - x_1)$

$$1^2 + 2^2 + 3(1)(2) - 11 = 0$$

$$1 + 4 + 6 - 11 = 0$$

$$11 - 11 = 0$$

$$y^2 + 3y = -x^2 + 11 - 3x$$

$$\frac{dy}{dx} = -2x + 11 - 3 = -1 + 11 - 3$$

$$\frac{dy}{dx} = 7 \quad m = 7$$

$$y + 3y = m(x - 1)$$

$$y + 6 = 1$$

$$y + 5 = 0$$