Maths

1. Velocity = $\frac{s}{t}$

Since we are dealing with position vectors

let $P(x, y, z)$ be any point on the curve

and $\vec{r} = x\hat{i} + y\hat{j}$ an $z\hat{k}$ be the position vector of P relative to O as the origin

Substituting x, y, z in $\vec{r}$

we have $\vec{r} = (7t)\hat{i} + (6t^2 + 4t)\hat{j} + (t-6)\hat{k}$

So velocity vector $\vec{V}$

will be differential of $\vec{r}$ in respect to t

$$\text{Vector } \vec{V} = \frac{dr}{dt} = 14t^{2}\hat{i} + (2 \times 6t^{2-1} - 4t^{1-1})\hat{j} + 1t^{1-1}$$

$$\vec{V} = \frac{dr}{dt} = 14t\hat{i} + (12t - 4)\hat{j} + \hat{k}$$

2 $A = 1 + 2j - 4k$

$B = 2i + Bj - k$

$C = 4j \; - 3k$

$A \times B \times C$

$1 + 2j - 4k \times [(2i - 3j + k) \times (4j - 3k)]$

$i + 2j - 4k \times [2i - 12j - 3k)$

$2i - 24j + 12k$

3 $R = 4\sin 3t\, i + 4e^{2t} j + 7t^3 k$

$$\int R\,dt = \frac{-4}{3}\cos 3t\, i + \frac{4}{3}e^{2t} j + \frac{7t^4}{4} k$$

4. $A = 7i + 2j - k$

$B = 2i + 3j + 4k$

$C = i + j + k$

$(A + C)\ (B - A)$

$[(7i + 2j - k) + (i + j + k)]$

$(8i + 3j)$

$[(2i + 3j + 4k) - (7i + 2j - k)]$

$(-5i - j + 5k)$

$(8i + 3j) \times (-5i - j + 5k)$

$= 40i - 3j$

5. $x = t \quad y = t^2 \quad z = t^3$

$r = xi + yj + zk$

$r = ti + t^2 j + tk^3$

$\frac{dr}{dt} = i + 2tj + 3t^2 k$

At $t = 1$ $\frac{dr}{dt} = i + 2(1)j + 3(1^2)k$

$\frac{dr}{dt} = i + 2j + 3k$

$|dr/dt| = \sqrt{1^2 + 2^2 + 3^2}$

$= \sqrt{14}$

$T = \frac{dr/dt}{|dr/dt|} = \frac{i + 2j + 3k}{\sqrt{14}}$