

Exercise 1: Vector Algebra

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1) $A = 2i - j$, $B = 7i - 11k$ and $C = 4j + 5k$
find the plane

$$-2A + 7B - 2C$$

2) $H = 2A + 2B - C$, find the direction cosine of $A \times (B \times C)$

$$3B \times B = (A \times B)$$

$$A \cdot 2B = C$$

Now perpendicular and explain with

Solution

$$1) -2(2i - j) - 2A = -6i + 3j$$

$$7B = 21i + 7j - 77k$$

$$3C = 32i + 30j - 40k$$

$$(-6i + 3j) + (21i + 7j - 77k)$$

$$= (15i + 10j - 77k) - (32i + 30j - 40k)$$

$$= -17i - 20j - 17k$$

$$2) H = 2A + 2B - C$$

$$2A = 4i - 2j$$

$$2B = 14i + 7j - 22k$$

$$C = 4j + 5k$$

$$\begin{aligned}
 \vec{r} &= 12\vec{i} - 2\vec{j} - 45\vec{k} \\
 |\vec{r}| &= \sqrt{(12)^2 + (-2)^2 + (-45)^2} \\
 &= \sqrt{144 + 4 + 2025} \\
 &= \sqrt{2549} \\
 &= 50.48
 \end{aligned}$$

\therefore direction cosine =

$$\frac{12}{50.48} = 0.237$$

$$\frac{-2}{50.48} = -0.039$$

$$\frac{-45}{50.48} = -0.89$$

$$\begin{aligned}
 \vec{A} \times (\vec{B} \times \vec{C}) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & -11 \\ 4 & 4 & -5 \end{vmatrix} \\
 &= \vec{i}(-15 - (-44)) - \vec{j}(-15 - (-44)) + \vec{k}(12 - 16) \\
 &= 29\vec{i} - 29\vec{j} + 8\vec{k}
 \end{aligned}$$

$$\vec{a} = 6\mathbf{i} - 3\mathbf{j}$$

$$\vec{b} = 6\mathbf{i} + 2\mathbf{j} - 22\mathbf{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$= \mathbf{i}(-66 - 0) - \mathbf{j}(-132 - 0) + \mathbf{k}(6 - (-12))$$

$$= -66\mathbf{i} + 132\mathbf{j} + 18\mathbf{k}$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 0 \\ 2 & 4 & 10 \end{vmatrix}$$

$$= \mathbf{i}(-30 - 0) - \mathbf{j}(60 - 0) + \mathbf{k}(24 - (-6))$$

$$= -30\mathbf{i} - 60\mathbf{j} + 30\mathbf{k}$$

$$8 (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) = (-66\mathbf{i} + 132\mathbf{j} + 18\mathbf{k}) \cdot (-30\mathbf{i} - 60\mathbf{j} + 30\mathbf{k})$$

$$= 1980 + (-7920) + 540 = -5400$$

$$5 \vec{a} - 2\vec{b} - \vec{c} = -8\mathbf{i} - 7\mathbf{j} - 17\mathbf{k}$$

$$2\mathbf{i} - \mathbf{j}$$

$$6\mathbf{i} + 2\mathbf{j} - 22\mathbf{k}$$

$$4\mathbf{j} + 4\mathbf{j} + 17\mathbf{k}$$

2. Vectors are said to be perpendicular if their dot product is equal to zero

" Vectors are said to be coplanar if their triple product is equal to zero