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MATRIC NO: 19/ENG04/015 SERIAL NO: 19
MAT104 ASSIGNMENT.

(1.) $\int \frac{11-3x}{x^2+2x-3}\,dx$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$A(x+3) + B(x-1) = 11-3x$

$Ax + 3A + Bx - B = 11-3x$

$A+B = -3 \rightarrow (1)$ $\quad 3A - B = 11 \rightarrow (2)$

$A = -3-B \rightarrow (3)$ $\quad 3(-3-B) - B = 11$ $\quad -9-3B-B = 11$

$-4B = 20$ $\quad B = -20/4 = -5$

$A = -3-(-5) = -3+5 = 2$

$\therefore \frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$

$= \int \frac{2}{x-1}\,dx - \int \frac{5}{x+3}\,dx$

Let $u = x-1$ $\quad v = x+3$

$du/dx = 1$ $\quad dv/dx = 1$

$2\int \frac{du}{u} - 5\int \frac{dv}{v}$

$= 2\ln|u| - 5\ln|v| + C$

$= 2\ln(x-1) - 5\ln(x+3) + C$

(2.) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2-9x-35$

$A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2) = 2x^2-9x-35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$

$A+B+C = 2 \rightarrow (1)$

$A+4B-C = -9 \rightarrow (2)$

$-6A+3B-2C = -35 \rightarrow (3)$

Solving simultaneously; $A = 4, B = -3, C = 1$

$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$

$4\int \frac{1}{x+1}\,dx - 3\int \frac{1}{x-2}\,dx + \int \frac{1}{x+3}\,dx$

let $u = x+1$ let $v = x-2$ let $w = x+3$

$du/dx = 1$ $dv/dx = 1$ $dw/dx = 1$

$= 4\int \frac{du}{u} - 3\int \frac{dv}{v} + \int \frac{dw}{w}$

$= 4\ln(u) - 3(\ln(v)) + \ln(w) + C$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$

13.) $\int \frac{1}{x^2+121}\,dx$

Given a right angled triangle, $\tan\theta = x/11$

$\therefore x = 11\tan\theta$ $dx/d\theta = 11\sec^2\theta$

~~Substituting $x = 11\tan\theta$~~ $dx = 11\sec^2\theta \cdot d\theta$

Substituting $x = 11\tan\theta$ and $dx = 11\sec^2\theta \cdot d\theta$

$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2 + 121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta + 121}$

Recall that; $1 + \tan^2\theta = \sec^2\theta$

$\therefore 121\tan^2\theta + 121 = 121\sec^2\theta$

$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$

$= \frac{1}{11}\left[\theta\right] + C$

If $\tan\theta = x/11$; $\theta = \tan^{-1}\left[x/11\right]$

$= \frac{1}{11}\tan^{-1}\left[\frac{x}{11}\right] + C$