

(6)

$f(x) = 3x^2 - 2x + 1 = 0$, Show that
 $f_0(x) + f_0(-x) = f$

$$f_0(x) = \frac{f(x) + f(-x)}{2}$$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$

$$f(-x) = 3x^2 + 2x + 1$$

$$f_0(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{6x^2 + 2}{2}$$

$$= \frac{1}{2}(3x^2 + 1)$$

$$\therefore f_0(x) = 3x^2 + 1$$

$$f_0(-x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$f_0(-x) = \frac{3x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$= \frac{-4x}{2} = -2x$$

$$\therefore f(x) = f_0(x) + f_0(-x)$$

$$f(x) = 3x^2 + 1 - 2x$$

$$= 3x^2 - 2x + 1$$

7) $y = \cos x$ from 1st Principle

$$y = \cos x$$

$$y + \theta y = \cos(x + \theta x)$$

Subtract y from both sides

$$\theta y = \cos(x + \theta x) - y$$

$$\text{but } y = \cos x$$

$$\therefore \theta y = \cos(x + \theta x) - \cos x \quad \text{--- (1)}$$

Consider from trigonometry

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) - \cos(A - B) = -2 \sin A \sin B \quad \text{--- (2)}$$

Compare (1) and (2)

$$\text{Let } A + B = x + \theta x \quad \text{--- (3)}$$

$$A - B = x \quad \text{--- (4)}$$

Adding (3) and (4)

$$2A = 2x + \theta x$$

$$A = \frac{2x + \theta x}{2} \Rightarrow A = x + \frac{\theta x}{2} \quad \text{--- (5)}$$

Substitute equ (5) in equ (3)

$$x + \left(\frac{\theta x}{2} - B\right) = x$$

$$B = \frac{\theta x}{2}$$

Compare equ (1) and (2)

$$\cos\left(x + \frac{\theta x}{2}\right) - \cos x = -2 \sin\left(x + \frac{\theta x}{2}\right) \sin\left(\frac{\theta x}{2}\right)$$

$$\therefore \theta y = -2 \sin\left(x + \frac{\theta x}{2}\right) \sin\left(\frac{\theta x}{2}\right)$$

$$\frac{\theta y}{\theta x} = -\sin\left(x + \frac{\theta x}{2}\right) \frac{\sin\left(\frac{\theta x}{2}\right)}{\frac{\theta x}{2}} \quad \text{--- (6)}$$

A standard limit

$$\lim_{\theta x \rightarrow 0} \frac{\sin\left(\frac{\theta x}{2}\right)}{\frac{\theta x}{2}} = 1$$

find limit of 4 as $\theta x \rightarrow 0$

$$\lim_{\theta x \rightarrow 0} \frac{\theta y}{\theta x} = \lim_{\theta x \rightarrow 0} -\sin\left(x + \frac{\theta x}{2}\right) \frac{\sin\left(\frac{\theta x}{2}\right)}{\frac{\theta x}{2}}$$

$$= -\sin(x + 0) \cdot 1$$

$$= -\sin x$$

$$\lim_{\theta x \rightarrow 0} \frac{\theta y}{\theta x} = \frac{dy}{dx} = -\sin x$$

$$8) y = 3t^2 \quad x = \frac{1}{t^2}$$

Solution

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{-2t}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{-2t} = -3$$

$$= -3$$

$$9) y = x^2 \cos(2x^2)$$

Using the Product rule

$$\frac{dy}{dx} = 2x \cos(2x^2) + x^2 (-\sin(2x^2) \cdot 4x)$$

Let $u = x^2$ and $v = \cos(2x^2)$

$$\frac{dv}{dx} = -2x \sin(2x^2)$$

$$v = \cos(2x^2)$$

Using the Chain rule

$$\frac{dy}{dx} = 2x \cos(2x^2) + x^2 (-\sin(2x^2) \cdot 4x)$$

$$= 2x \cos(2x^2) - 4x^3 \sin(2x^2)$$

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$$\frac{dy}{dx} = \cos(2x^2) \cdot 2x + x^2 (-\sin(2x^2) \cdot 4x)$$

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Maths 101 Assignment

1) Function $y = 1/x - 2$
Solution

The function is defined for all numbers except $x=2$

Domain = Real numbers except $x=2$

Co-Domain = Real numbers except $y=0$

2) If $k = \ln v$, differentiate k
 $\frac{d}{dx} (\ln v) = \frac{1}{v}$

$$\cos^2 p = 1 - \sin^2 p$$

$$\cos p = \sqrt{1 - \sin^2 p}$$

$$\cos p = \sqrt{1 - t^2}$$

$$\frac{dt}{dp} = \cos p = \sqrt{1 - t^2}$$

$$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$$

3) a) $2x - 3y - 2 = 0$
 $2x - 2 = 3y$
 $y = \frac{2x + 2}{3}$

b) $x^2 + y^2 = 4$
 $x^2 - 4 = -y^2$
 $y = \pm \sqrt{4 - x^2}$

5a) $(f \circ g)(x) = f(g(x))$

$$f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$f(g(x)) = f(4x - 2)$$

$$= 2(4x - 2)^2 - 5$$

$$= (8x - 4)^2 - 5$$

$$= 64x^2 - 32x + 16 - 5$$

$$= \underline{64x^2 - 32x + 11}$$

4) If $p = \sin^{-1} t$ find the derivative of p .

$$p = \frac{t}{\sin}$$

$$t = \sin p \quad \dots \dots \textcircled{1}$$

Recall that; $\sin^2 p + \cos^2 p = 1 \dots \textcircled{2}$

$$\frac{dt}{dp} \text{ of } (1) = \cos p$$

b) $(g \circ f)(x)$

$$g[f(x)] = g(2x^2 - 5)$$

$$= 4x - 2 [2x^2 - 5]$$

$$= 4(2x^2 - 5) - 2$$

$$= [8x^2 - 20] - 2$$

$$= 8x^2 - 20 - 2$$

$$= \underline{8x^2 - 22}$$

from (2) $\sin^2 p + \cos^2 p = 1$