ADEMIRAN MUSLADEEMAT ADEDAMOLA
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COMPUTER ENGINEERING

Differentiate the following

1.) $y = \dfrac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x-3)^{4/3}]}$

$\ln y = \ln[(x+1)^2] + \ln(\sqrt{x-2}) - \ln(2x-1) - \ln[(x+3)^{3/2}]$

$(1/y)\, dy/dx = \dfrac{1}{(x+1)^2} \cdot 2(x+1) + \dfrac{1}{\sqrt{x-2}} \cdot 1/2 (x-2)^{-1/2} - \dfrac{1}{2x-1} \cdot 2$

$- \dfrac{1}{(x+3)^{3/2}} \cdot \dfrac{3}{2}(x+3)^{1/2}$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{2}{(x+1)} + \dfrac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \dfrac{2}{2x-1} - \dfrac{3}{2}(x+3)^{1/2}$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{x+1} + \dfrac{1}{2(x-2)} - \dfrac{2}{2x-1} - \dfrac{3}{2(x+3)}\right]$

$\dfrac{dy}{dx} = \dfrac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x+3)^{3/2}}\left[\dfrac{2}{x+1} + \dfrac{1}{2x-4} - \dfrac{2}{2x-1} - \dfrac{3}{2x+6}\right]$

2.) $y = \dfrac{3e^x \sin 2x}{x^{5/2}}$

$\ln y = \ln(3e^x) + \ln(\sin 2x) - \ln(x^{5/2})$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{1}{3e^x} \cdot 3e^x + \dfrac{1}{\sin 2x} \cdot 2\cos 2x - \dfrac{1}{x^{5/2}} \cdot \dfrac{5}{2} x^{3/2}$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = 1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5}{2} x^{3/2 - 5/2}$

$\dfrac{dy}{dx} = y\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5x^{-1}}{2}\right]$

$\dfrac{dy}{dx} = y\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5x^{-1}}{2}\right]$

$\dfrac{dy}{dx} = \dfrac{3e^x \sin 2x}{x^{5/2}}\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5}{2x}\right]$

Integrate the following with respect to its variable

1) $4\sec^2(3m+1)$

$4\int \sec^2(3m+1)\,dm$

$u = 3m+1$

$\frac{du}{dm} = 3$

$du = 3\,dm$

$dm = \frac{du}{3}$

$4\int \sec^2(u)\,du$

$\frac{4}{3}\tan u + C$

$= \frac{4}{3}\tan(3m+1) + C$

2) $\int 2t(3t^2-1)^{1/2}\,dt$

$u = \sqrt{3t^2-1}$

$u^2 = 3t^2-1$

$3t^2 = u^2+1$

$t^2 = \frac{u^2+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$\int 2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$\frac{2}{3}\int u^2\left(\frac{u^2+1}{3}\right)^{1/2-1/2} du$

$= \frac{2}{3}\int u^2\,du$

$= \frac{2}{3}\left[\frac{u^3}{3}\right] + c$

$= \frac{2u^3}{9} + c$

$= \frac{2(3t^2-1)^{3/2}}{9} + c$

5.) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$u = \sqrt{4x^2-1}$

$u^2 = 4x^2 - 1$

$4x^2 = u^2 + 1$

$x^2 = \frac{u^2+1}{4}$

$x = \sqrt{\frac{u^2+1}{4}}$

$= \frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$

$= \frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$= dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$= \int 2\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$= \frac{1}{2}\int\left(\frac{u^2+1}{4}\right)^{1/2-1/2} du$

$= \frac{1}{2}\int du$

$= \frac{u}{2} + c$

$= \frac{\sqrt{4x^2-1}}{2} + c$