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19/ENG06/042 Mechanical engineering 046

1.) $\int \frac{11-3x}{x^2+2x-3}$ change into partial fraction.

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x+3)(x-1)} \Rightarrow \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\Rightarrow \frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

multiply both sides by $(x+3)(x-1)$

$\Rightarrow 11-3x = A(x-1)+B(x+3)$

When $x = 1$

$11-3 = 4B$

$\Rightarrow 4B = 8 \quad \therefore B = 2.$

When $x = -3.$

$11-3(-3) = A(-3-1)+B(-3+3)$

$\Rightarrow 20 = -4A \quad \therefore A = -5.$

$$\Rightarrow \frac{-5}{(x+3)} + \frac{2}{(x-1)}.$$

$$\therefore \int \frac{-5}{x+3}\,dx + \int \frac{2}{(x-1)}\,dx$$

let $u_1 = x+3$ $\quad$ let $u_2 = x-1$

$du/dx = 1$ $\quad$ $du/dx = 1$

$dx = du$ $\quad$ $du = dx$

$$\int \frac{-5\,du}{u} + \int \frac{2\,du}{u}$$

$\Rightarrow -5\ln u_1 + 2\ln u_2 + C$

$\Rightarrow -5\ln(x+3) + 2\ln(x-1) + C$

2.) $\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$

$$\frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$\therefore \frac{A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)}{(x+1)(x-2)(x+3)}$$

$\therefore 2x^2-9x-35 = Ax^2+Ax-6A + Bx^2+4Bx+3B + Cx^2+Cx-2C.$

$\therefore (A+B+C)x^2 + (A+4B-C)x + (6A+3B-2C) = 2x^2-9x-35.$

$A+B+C = 2 \rightarrow (1)$
$A+4B-C = -9 \rightarrow (2)$
$-6A+3B-2C = -35 \rightarrow (3)$.

{ Make A subject of formula in equation (1) and subs. in eqn (2) and (3). }

$A = 2-B-C \rightarrow (4)$.

$\Rightarrow 2-B-C+4B-C = -9$

$\Rightarrow 3B-2C = -11 \rightarrow (5)$

$\Rightarrow -6(2-B-C)+3B-2C = -35$.

$\Rightarrow 9B+4C = -23 \rightarrow (6)$.

$\therefore$ Make C subject of formula in eqn (5).

$C = \frac{3B+11}{2}$ --- (7)

Substitue eqn. 7 into equation (6).

$9B + {}^{4}/_{2}(3B+11) = -23$

$9B + 2(3B+11) = -23 \Rightarrow 9B+6B = -23-22$

$\Rightarrow 15B = -45 \quad \therefore B = -3$.

Substitute -3 for B in equation (7)

$$C = \frac{3B + 11}{2} \Rightarrow \frac{3(-3) + 11}{2}$$

$$C \Rightarrow \frac{2}{2} \Rightarrow 2.$$

Substitute B and C for -3 and 2 respectively

$$A + B + C = 2 \quad - (1)$$

$$A - 3 + 1 = 2$$

$$\therefore A = 2 + 3 - 1 \quad \therefore A = 4.$$

$$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \Rightarrow \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{x+3}.$$

$$\therefore \int \frac{4}{(x+1)}\,dx + \int \frac{-3}{(x-2)}\,dx + \int \frac{1}{(x+3)}\,dx.$$

Let $u_1 = x+1$	let $u_2 = x-2$	Let $u_3 = x+3$
$du/dx = 1$	$du_2/dx = 1$	$du_3/dx = 1$
$dx = du_1$	$dx = du_2$	$dx = du_3$.
$\int \frac{4\,du_1}{u_1}$	$\int \frac{-3\,du_2}{u_2}$	$\int \frac{du_3}{du}$

$$\Rightarrow 4\ln(u_1) - 3\ln(u_2) + \ln(u_3).$$

$$\Rightarrow 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$

3.) $\int \frac{1}{(x^2+121)}\,dx$ ~~$\Rightarrow u = x^2 + 121$~~

~~$\therefore du/dx = 2x$~~

$$\Rightarrow \int \frac{dx}{x^2 + 11^2} \Rightarrow \frac{1}{x}\left(\tan^{-1}\frac{11}{x}\right) + C$$