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MATRIC NO: 19/MHS11/137.
MTH 104

(1) For what values of x is the function $y = 1/(x-2)$ defined? State the domain and co-domain.

Ans: The function is defined for all real numbers except $x = 2$

Domain = Real numbers except $x = 2$

Codomain = Real numbers except $y = 0$.

(2) If $k = \ln 4$, differentiate k

$k = \ln 4$

$dk = \frac{1}{4}$

(3) Express y and an explicit function of $\ln x$ in the following

(a) $2x - 3y - 2 = 0$

$-3y = 2 - 2x$

$y = \frac{2-2x}{-3}$ $y = 2x - 2/3$

$y = 2x - 2/3$ //

(b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

(4) Find dp/dt, $p = \sin^{-1} t$

$p = \frac{t}{\sin}$; $t = \sin p$

$\frac{dt}{dp} = \cos p$; $\frac{dp}{dt} = 1/\cos p$

Recall that $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm\sqrt{1 + \sin^2 y}$

$t = \sin p$

$\therefore \cos p = \sqrt{1 + t^2}$

Hence, $dp/dt = 1/(1 - t^2)$

(5) If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $f \circ g(x)$ and $g \circ f(x)$

$f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$f \circ g(x) = 2(4x - 2)^2 - 5$

$2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

(α). $3x^2 - 2x + 1 = 0$, show that $f_e(\alpha) + f_o(\alpha) = f$

$f(x) = 3x^2 - 2x + 1$

$f_e(a) = f(a) + f(-a)$

$f(-x) = 3(-x)^2 - 2(-x) + 1$
$= 3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$
$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f(a) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2} = -\frac{4x}{2} = -2x$

$f(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$f_e(x) + f_o x = 3x^2 \cancel{+1 + 1 - 2x} n$
$= 3x^2 - 2x + 1$

(2) Differentiate $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ — (1) $y = \cos x$

Recall

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — (2)

Comparing (1) & (2)

$A + B = x + \delta x$ — (3)

$A - B = x$ — (4)

Adding (3) and (4) and substracting (3) and (4)

$2A = 2x + \delta x$ & $B = \delta x/2$

$A = x + \delta x/2$

Comparing (1) and (2)

$\delta y = \cos(x + \delta x) - \cos x$

$-2\sin(x + \delta x/2)\sin(\delta x/2)$

dividing through by δx

$\frac{\delta y}{\delta x} = -2\sin(x + \delta x/2)\sin($

$\frac{\delta y}{\delta x} = \frac{-\sin(x + \delta x/2)\sin(\delta x/2)}{\delta x/2}$

(b) $y = \sin(3x^3+5)$

Let $u = 3x^3+5$

$\frac{dy}{du} = \cos u$

$\frac{du}{dx} = 9x^2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 9x^2$

$= 9x^2 \cos u$

$= 9x^2 \cos 3x^3+5$

$(\cos x) = \phi$

$= -\sin(x + \delta x/2) \times \frac{\sin(\delta x/2)}{\delta x/2}$

Taking limits $\delta x \to 0$

$\lim_{\delta x \to 0} \frac{\sin \delta x/2}{\delta x/2} = 1$

$\delta y/\delta x = -\sin(x + 0/2) \times 1$

$\delta x \to 0$

$\delta y/\delta x = -\sin x$

(8) $y = 3t^2$; $x = 1/t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dy}{dt} = 6t$; $\frac{dx}{dt} = \frac{-2}{t^3}$

$\frac{dy}{dx} = 6t \div \frac{-2}{3}$

$= 6t \times \frac{-2}{t^3} = \frac{6 \times -2}{t^2} = \frac{-12}{t^2}$

$\frac{dy}{dx} = \frac{-12}{t^2}$

(9) $y = x^2 \cos 2x \, e^{4x}$

Solution

Taking $\log_e$ of both sides

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

Differentiating both with x

$\frac{1}{y} = \frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x)$

$+4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

Multiplying both sides by y

$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$

$= x^2 \cos 2x \, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x}$