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Computer Engineering -

19/ENG021006.

Maths 104 assignment -

Integrate the following with respect to their variable.

$$\int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)}{(x-1)(x+3)} + \frac{B(x-1)}{(x-1)(x+3)}$$

multiply both sides by  $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

At  $x = -3$ , we have

$$B(-3-1) = 11-3(-3)$$

$$-4B = 20$$

$$B = -5$$

At  $x = 1$ , we have

$$A(1+3) = 11-3(1)$$

$$4A = 8$$

$$A = 2$$

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\int \frac{2dx}{x-1} + \int \frac{-5dx}{x+3} = \int \frac{11-32x}{x^2+2x-3} dx$$

$$2 \int \frac{dx}{x-1} \quad -5 \int \frac{dx}{x+3}$$

$$u = x-1$$
$$\frac{du}{dx} = 1$$

$$u = x+3$$
$$\frac{du}{dx} = 1$$

$$dx = du$$

$$dx = du$$

$$2 \int \frac{du}{u}$$

$$-5 \int \frac{du}{u}$$

$$= 2 \ln u$$

$$-5 \ln u \Rightarrow 2 \ln(x-1) - 5 \ln(x+3)$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = 2 \ln(x-1) - 5 \ln(x+3)$$

$$2x^2 - 9x - 35 = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$2x^2 - 9x - 35 = A(x^2 + xc - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A+B+C=2 \quad \text{--- (1)}$$

$$A+4B-C=9 \quad \text{--- (2)}$$

$$-6A+3B-2C=-35 \quad \text{--- (3)}$$

$$\text{from (1) } A=2-B-C \quad \text{--- (4)}$$

Put (4) into (2) and (3)

$$(2-B-C) + 4B - C = 9$$

$$4B - B - C - C = 9 - 2$$

$$3B - 2C = -11$$

$$3B - 2C = -11 \times 3$$

$$15B + 4C = -23 \times 1$$

$$15B - 6B = -33$$

$$-15B + 4C = -23$$

$$+4C = -10$$

$$C = 1$$

$$15B + 4C = -23$$

$$15B = -23 - 4$$

$$15B = -27$$

$$B = -\frac{27}{15}$$

$$3B - 2 = -11$$

$$3B = -11 + 2$$

$$3B = -9$$

$$B = -3$$

$$A = 2 - (-3) - 1$$

$$A = 2 + 3 - 1 = 5 - 1$$

$$A = 4$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \int \frac{4 dx}{x+1} + \int \frac{-3 dx}{x-2} + \int \frac{dx}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \int \frac{dx}{x+1} - 3 \int \frac{dx}{x-2} + \int \frac{dx}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \begin{array}{ccc} u = x+1 & u = x-2 & u = x+3 \\ \frac{du}{dx} = 1 & \frac{du}{dx} = 1 & \frac{du}{dx} = 1 \end{array}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \int \frac{du}{u} - 3 \int \frac{du}{u} + \int \frac{du}{u}$$

$$= 4 \ln(x)$$

$$= 4 \ln u - 3 \ln u + \ln u$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$

$$\int \frac{1}{x^2+121} dx = \int \frac{dx}{x^2+121} = \int \frac{dx}{x^2+11^2}$$

$$x = 11 \tan \theta$$

$$dx/d\theta = 11 \sec^2 \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2+11^2 = 11^2 \sec^2 \theta + 11^2 = 11^2 ($$

$$x^2+11^2 = 11^2 \tan^2 \theta + 11^2 = 11^2 (\tan^2 \theta + 1)$$

$$\text{recall that } 1 + \tan^2 \theta = \sec^2 \theta$$

$$= 121 \sec^2 \theta$$

$$\Rightarrow \int \frac{11 \sec^2 \theta d\theta}{121 \sec^2 \theta} = \int \frac{d\theta}{11} = \frac{1}{11} \int d\theta$$

$$= \frac{1}{11} (\theta) + C$$

$$\text{and } x = 11 \tan \theta$$

$$\text{or } \tan \theta = \frac{x}{11}$$

$$\theta = \tan^{-1} \frac{x}{11}$$

$$\int \frac{1}{x^2+121} dx = \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$