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Course: MAT 104 Assignment.

1. For what values of x is the function $y = \frac{1}{(x-2)}$ defined? State the domain and codomain.

Soln:

The function $y = \frac{1}{(x-2)}$ is defined for all real numbers except $x = 2$.
The domain is the set of real numbers except $x=2$
The codomain is the set of real numbers except $y=0$

2. If $K = \ln v$, differentiate K.

Soln

$K = \ln v$

$\frac{dK}{dx} = \frac{1}{v}$

or v^{-1}

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$\frac{dy}{dx} = 9x^2 \cos u$$

but $u = 3x^3 + 5$

$$\therefore \frac{dy}{dx} = 9x^2 \cos(3x^3 + 5)$$

9. $y = x^2 \cos 2x \, e^{4x}$

find $\log_e$ of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

differentiating both sides with respect to x

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2} + \frac{-2\sin 2x}{\cos 2x} + \frac{4e^{4x}}{e^{4x}}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + \frac{4e^{4x}}{e^{4x}}$$

$= \cancel{3x}\ 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$g \circ f(x) = g(f(x))$

$= g(2x^2 - 5)$

$= 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

6. If $f(x) = 3x^2 - 2x + 1 = 0$. Show that $f_e(x) + f_o(x) = f(x)$

Sol^n.

$f_e(x) + f_o(x) = f(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \dfrac{f(x) + f(-x)}{2}$ $\quad f_o(x) = \dfrac{f(x) - f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$f_e(x) = \dfrac{3x^2 - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2}$

$= \dfrac{6x^2 + 2}{2}$

$= \cancel{2}\left(\dfrac{3x^2 + 1}{\cancel{2}}\right)$

$\frac{dt}{dp}$ of $(t) = \cos p$

from (2) $\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin p}$

$\cos p = \sqrt{1 - t^2}$

$\frac{dt}{dp} = \cos p = \sqrt{(1-t)^2}$

$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$ //

5. If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $fog(x)$ and $gof(x)$.

Soln.

$fog(x) = f(g(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$f(g(x)) = f(4x - 2)$

$= 2(4x - 2)^2 - 5$

$= 2(4x-2)(4x-2) - 5$

$= 2(16x^2 - 8x - 8x + 4) - 5$

$= 2(16x^2 - 16 + 4) - 5$

$f_e(x) = 3x^2 + 1$

$$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$= \frac{\cancel{3x^2} - 2x \cancel{+1} - \cancel{3x^2} - 2x \cancel{-1}}{2}$$

$$= \frac{-4x}{2} = -2x$$

$f(x) = f_e(x) + f_o(x)$

$= 3x^2 + 1 + (-2x)$

$= 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7. $y = \cos x$

$y \pm dy = \cos(x + \delta x)$

Subtract y from both sides

$dy = \cos(x + \delta x) - y$

but $y = \cos x$

$dy = \cos(x + \delta x - \cos x) \ldots$ (*)

Consider from trigonometry

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ (2*)

$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B \ldots$

3. Express y as an explicit function of x in the following.

a. $2x - 3y - 2 = 0$

b. $x^2 + y^2 = 4$

Soln:

a. $2x - 3y - 2 = 0$

$$\frac{2x-2}{3} = \frac{\cancel{3}y}{\cancel{3}}$$

$$\frac{2x-2}{3} = y \qquad \therefore y = \frac{2x-2}{3}$$

b. $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

Take square root of both sides

$y = \pm\sqrt{4 - x^2}$

4. If $P = \sin^{-1} t$, find the derivative of P

Soln

$$P = \frac{t}{\sin}$$

$t = \sin P \quad \ldots (1)$

Recall that, $\sin^2 P + \cos^2 y = 1 \quad \ldots (2)$

multiply both sides by y

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + \frac{4e^{4x}}{e^{4x}}\right)$$

$$\frac{dy}{dx} = x^2 \cos 2x\, e^{4x}\left(\frac{2}{x} - 2\tan 2x + \frac{4e^{4x}}{e^{4x}}\right)$$

$= \cancel{32x} \; 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$ //

$g \circ f(x) = g(f(x))$

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6. If $f(x) = 3x^2 - 2x + 1 = 0$. Show that $f_e(x) + f_o(x) = f(x)$

Soln.

$f_e(x) + f_o(x) = f(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \dfrac{f(x) + f(-x)}{2}$ $\quad f_o(x) = \dfrac{f(x) - f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$f_e(x) = \dfrac{3x^2 \cancel{4} - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2}$

$= \dfrac{6x^2 + 2}{2}$

$= \cancel{2}\left(\dfrac{3x^2 + 1}{\cancel{2}}\right)$

Compare (*) and (2*)

Let

$A + B = x + \delta x$ . . . (i)

$A - B = x$ . . . (ii)

Adding equation (i) and (ii) together

$2A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$$\left.\begin{aligned} A &= x + \frac{\delta x}{2} \\ B &= \frac{\delta x}{2} \end{aligned}\right\} (3*)$$

Eqn (*)

Compare (*) and (2*)

$\cos(x + \delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\therefore \delta y = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$

$\frac{\delta y}{\delta x} = \frac{-\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$

$\frac{\delta y}{\delta x} = -\sin\left(x + \frac{\delta x}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$ . . . (4*)

A standard limit

$$\lim_{\delta x \to 0} \frac{\sin\left(\frac{\delta x}{2}\right)}{\delta x/2} = 1$$

find limit of (4*) as $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin\left(x + \frac{\delta x}{2}\right) \frac{\sin\left(\frac{\delta x}{2}\right)}{\delta x/2}$$

$$= -\sin(x + 0 \cdot 1$$

$$= -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

8. $y = 3t^2$ $x = 1/t^2$. find dy/dx

$y = 3t^2$: $dy/dt = 6t$

$x = 1/t^2$: $dx/dt = -2t^{-3}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{-2\frac{1}{t^3}}$$

$$= 6t \times -2t^{-3} \qquad = -12t^{-2} \quad \text{or} \quad -12\frac{1}{t^2}$$

10. ~~$y = x^2 \cos 2x e^{4x}$~~ $y = \sin(3x^3 + 5)$

let $u = 3x^3 + 5$: $du/dx = 9x^2$

$\Rightarrow$ $y = \sin u$: $dy/du = \cos u$