ASSIGNMENT

1. A particle moves along a curve $x = 7t^2$, $y = 8t^2 - 4t$, $z = t - 5$, where t is time. Find it's velocity.

Solution.

$x = 7t^2$, $y = 6t^2 - 4t$ and $z = t - 5$

$r = xi + yj + zk$

$r = (7t^2)i + (6t^2 - 4t)j + (t-5)k$

$$\text{Velocity} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = 14ti + (12t-4)j + k$$

2. If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$. Find $A \times (B \times C)$

Solution

$A = i + 2j - 4k$

$B = 2i - 3j + k$

$C = 4j - 3k$

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$(B \times C) = i\begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= i(9-4) - j(-6-0) + k(8-0)$$

$$= 5i + 6j + 8k$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A \times (B \times C) = i\begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j\begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$A \times (B \times C) = i(16+24) - j(8+20) + k(10-6)$

$= 40i - 28j + 4k$

3. Given $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$, find the integral of R with respect to t.

Solution

$R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

$\int R = \int 4\sin 3t\, i + \int 4e^{3t} j + \int 7t^3 k$

$\int R = 4i \int \sin 3t + 4j \int e^{3t} + 7k \int t^3$

$\int R = 4i \frac{(-\cos(3t))}{3} + 4j \frac{e^{3t}}{3} + 7k \frac{t^4}{4}$

$\int R = \frac{-4\cos(3t) i}{3} + \frac{4e^{3t} j}{3} + \frac{7t^4 k}{4}$

4. If $A = 7i + 2j - k$; $B = 2i + j + 4k$, $C = i + j + k$, find $(A+C) \cdot (B-A)$

Solution

$A = 7i + 2j - k$

$B = 2i + j + 4k$

$C = i + j + k$

$(A+C) \cdot (B-A) = [(7i+2j-k) + (i+j+k)] \cdot [(2i+j+4k) - (7i+2j-k)]$

$= [8i + 3j + 0k] \cdot [-5i - j + 5k]$

$= -40 - 3 + 0$

$= -43$

(5.) Find a unit vector tangent to the space curve $x=t$, $y=t^2$, $z=t^3$ at the point where $t=1$.

Solution

$$T = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$$

$$r = xi + yj + zk$$

$$r = ti + t^2 j + t^3 k$$

$$\frac{dr}{dt} = i + 2tj + 3t^2 k$$

at $t=1$, $\frac{dr}{dt} = i + 2tj + 3t^2 k$

$$= i + 2(1)j + 3(1)^2 k$$

$$= i + 2j + 3k$$

$$\left|\frac{dr}{dt}\right|_{u=1} = \sqrt{(i)^2 + (2j)^2 + (3k)^2}$$

$$= \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{1+4+9}$$

$$= \sqrt{14} = 3.74$$

Hence $T = \frac{i + 2j + 3k}{3.74}$