3 $\int \frac{2x}{(4x^2-1)^{1/2}} = \int 2x(4x^2-1)^{1/2}\, dx$

$u = 4x^2 - 1$

$du = 8x\,dx$

$dx = \frac{du}{8x}$

$= \int 2x(u)^{-1/2} \frac{du}{dx}$

$= 1/4 \int u^{-1/2}\, du$

$= 1/4 \times \frac{u^{-1/2+1}}{-1/2+1}$

$= \frac{1}{4} \times \frac{u^{1/2}}{1/2}$

$= 1/4 \times 2u^{1/2}$

$= 1/2\, u^{1/2} = 1/2(4x^2-1)^{1/2}$

1 $\int 4\sec^2(3m+1)\,dm$

$u = 3m+1$

$du = 3\,dm$

$dm = \frac{du}{3}$

$\int 4\sec^2 u \frac{du}{3}$

$4/3 \int \sec^2 u\,du$

$4/3 \tan u + c$

$4/3 \tan(3m+1) + C_2$

2 $\int 2t \times (3t^2 - 1)^{1/2}$

$u = 3t^2 - 1$

$\frac{du}{dt} = \frac{6t}{6t} dt$

$dt = \frac{du}{6t}$

$\int 2t \times (u)^{1/2} \frac{du}{3}$

$\int \frac{1}{3} \times u^{1/2} du$

$1/3 \int u^{1/2} du$

$= 1/3 \times \frac{u^{1/2+1}}{1/2+1} + C$

$= 1/3 \times 2/3 u^{3/2} + C$

$= 2/9 u^{3/2} + C$

$= 2/9 (3t^2 - 1)^{3/2} + C_2$

$$y = \frac{3e^x \sin^2 x}{x^{3/2}}$$

$$\ln y = \ln 3e^x + \ln \sin^2 x - \ln x^{5/2}$$

Differentiating with respect to x

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3e^x) + \frac{d}{dx}(\ln \sin x) - \frac{d}{dx}(\ln x^{5/2})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3e^x}(3e^x) + \frac{1}{\sin 2x}(\cos 2x) - \frac{1}{x^{5/2}}\left(\frac{5}{2}x^{3/2}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3e^x}{3e^x} + \frac{\cos 2x}{\sin 2x} - \frac{5/2x^{3/2}}{x^{5/2}}$$

multiply both sides by y

$$\frac{1}{y}\frac{dy}{dx} \times y = y\left(\frac{3e^x}{3e^x} + \frac{\cos 2x}{\sin 2x} - \frac{5/2x^{3/2}}{x^{5/2}}\right)$$

$$\frac{dy}{dx} = y\left(1 + \frac{\cos 2x}{\sin 2x} - \frac{5/2x}{x^{3/2}}\right)$$

$$\frac{dy}{dx} = \frac{3e^x \sin 2x}{x^{5/2}}\left(\frac{1 + \cos 2x}{\sin 2x} - \frac{5/2x^{3/2}}{x^{3/2}}\right)$$

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A

1 $y = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{3/2}}$

$\ln y = [\ln(x+1)^2 + \ln(x-2)^{1/2}] - [\ln(2x-1) + \ln(x-3)^{3/2}]$

$\frac{1}{y}\cdot\frac{dy}{dx} = \left[\frac{1}{(x+1)^2}\cdot 2(x+1) + \frac{1}{(x-2)^{1/2}}\cdot\frac{(x-2)^{-1/2}}{2}\right] - \left[\frac{1}{2x-1}\cdot 2 + \frac{1}{(x-3)^{3/2}}\cdot\frac{3(x-3)^{1/2}}{2}\right]$

$\frac{1}{y}\cdot\frac{dy}{dx} = \left[\frac{2(x+1)}{(x+1)^2} + \frac{(x-2)^{-1/2}}{2(x-2)^{1/2}}\right] - \left[\frac{2}{2x-1} + \frac{3(x-3)^{1/2}}{2(x-3)^{3/2}}\right]$

$\frac{1}{y}\cdot\frac{dy}{dx} = \left[\frac{2}{(x+1)} + \frac{1}{2(x-2)}\right] - \left[\frac{2}{2x-1} + \frac{3}{2(x-3)^2}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x-3)^2}\right]$