9th of April, 2020

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DEPARTMENT: ELECTRICAL ELECTRONICS ENGINEERING

MATRIC NO: 19/ENG04/042

SERIAL NO: 106

Integrate the following with respect to their variable

1) $\frac{(11-3x)\,dx}{x^2+2x-3}$ 2) $\frac{(2x^2-9x-35)\,dx}{(x+1)(x-2)(x+3)}$ 3) $\frac{1\,dx}{x^2+121}$

1) $\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

$= \frac{Ax-A+Bx+3B}{(x+3)(x-1)}$ $\therefore Ax-A+Bx+3B = 11-3x$

$A+B=-3$ — — ①

$-A+3B=11$ — — ①

from eqn ① $A = 3B-11$

$\therefore 3B-11+B=-3$

$4B=8 \quad \therefore B=2$

from eqn ① $A+B=-3$

$A=-3-B$

where $B=2$ $\quad A=-3-2=-5$

$\therefore \int \frac{-5}{x+3}\,dx + \int \frac{2}{x-1}\,dx = \int \frac{11-3x}{x^2+2x-3}\,dx$

Let $u=x+3$ $\qquad$ let $u=x-1$

$du/dx=1 \therefore dx=du$ $\qquad du/dx=1 \therefore dx=du$

$\Rightarrow \int \frac{-5}{u}\,du \qquad \Rightarrow \int \frac{2}{u}\,du$

$-5\int \frac{du}{u} \qquad 2\int \frac{du}{u}$

$= -5\ln u \qquad = 2\ln u$

$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = -5\ln(x+3) + 2\ln(x+1)$

$= -5\ln(x+3) + 2\ln(x-1) + C$

2) $\frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}\,dx = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$= \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)}{(x+1)(x+3)(x-2)}$

$= \frac{A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)}{(x+1)(x-2)(x+3)}$

$= \frac{Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C}{(x+1)(x-2)(x+3)}$

$\therefore Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C = 2x^2-9x-35$

$A+B+C = 2 \quad --(i)$

$A+4B-C = -9 \quad ---(ii)$

$-6A+3B-2C = -35 \quad ---(iii)$

@ eqn (i) Where $A+B+C=2$

Let $C = 2-A-B$

@ eqn (ii) $A+4B-(2-A-B) = -9$

$A+4B-2+A+B = -9$

$2A+5B = -7 \quad --(iv)$

@ eqn (iii) $-6A+3B-2(2-A-B) = -35$

$-6A+3B-4+2A+2B = -35$

$-4A+5B = -31 \quad ---(v)$

Solving eqn (iv) and (v) simultaneously

$2A + 5B = -7 \quad ---(iv)$

$-4A + 5B = -31 \quad ---(v)$

multiply eqn (iv) by 2

$\therefore 4A + 10B = -14 \quad --(iv)$

$-4A + 5B = -31 \quad --(v)$

$0 + 15B = -45$

$15B = -45$

$\therefore B = -3$

Using eqn (iv)

$2A + 5B = -7$ where $B = -3$

$2A + 5(-3) = -7$

$2A - 15 = -7$

$2A = 8$

$A = 4$

RECALL $C = 2 - A - B$; where $A = 4, B = -3$

$\therefore C = 2 - 4 + 3$

$C = 1,$

$$\therefore \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx = \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx$$

Let $u = x+1$ | Let $u = x-2$ | Let $u = x+3$

$du/dx = 1 \therefore dx = du$ | $du/dx = 1.$ | $du/dx = 1$

$\Rightarrow \int \frac{4}{u}\,du$ | $dx = du$ | $dx = du$

$4\int \frac{du}{u}$ | $\Rightarrow \int \frac{-3}{u}\,du$ | $\Rightarrow \int \frac{du}{u}$

$= 4\ln u$ | $= 3\int \frac{du}{u} = -3\ln u$ | $= \ln u$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$

3) $\int \frac{1}{x^2+121}\,dx$

With a right angle triangle; $\tan\theta = x/11$

$\therefore x = 11\tan\theta$

$\frac{dx}{d\theta} = 11\sec^2\theta$

$dx = 11\sec^2\theta \cdot d\theta$

Substituting $x = 11\tan\theta$ and $dx = 11\sec^2\theta \cdot d\theta$

$$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2+121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta+121}$$

Recall that: $1 + \tan^2\theta = \sec^2\theta$

$\therefore 121\tan^2\theta + 121 = 121\sec^2\theta$

$$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11}$$

$$= \frac{1}{11}\int d\theta = \frac{1}{11}[\theta] + C$$

Since $\tan\theta = x/11$; $\theta = \tan^{-1}[x/11]$

$$= \frac{1}{11}\tan^{-1}\left[\frac{x}{11}\right] + C$$