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19/MHS11/097
PHARMACY

MAT 104

1. $y = \frac{1}{x-2}$

- The function is defined for all real numbers except $x=2$
- The domain is the set of real numbers except $x=2$
- The codomain is the set of real number except $y=0$

2. $K = \ln V$

$\frac{dK}{dv} = \frac{1}{v}$

3. a) $2x - 3y - 2 = 0$

$-3y = 2 - 2x$

$y = \frac{2-2x}{-3}$

$y = \frac{2x+2}{3} ; \frac{2(x+1)}{3}$

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

4. find dP/dt, $P = \sin^{-1}t$

$P = \frac{t}{\sin}$; $t = \sin p$ → *

$\frac{dt}{dp} = \cos p$; $\frac{dP}{dt} = \frac{1}{\cos p}$

Recall, $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm\sqrt{1 \cancel{+} \sin^2 y}$

~~Comparing with Eq (*)~~ $t = \sin p$

$\therefore \cos p = \sqrt{1-t^2}$

$\boxed{\text{Hence, } dP/dt = 1/(1-t^2)}$

5. $f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$fog(x) = 2(4x-2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$gof(x) = 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22.$

6) Show that $f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= -4x/2 = -2x$

$f_e(x) + f_o(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ — (1) $(y = \cos x)$

Recall,

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — (2)

Comparing (1) & (2)

~~$A + B = x + \delta x$~~ — (3)

$A - B = x$ — (4)

Adding (3) & (4) & subtracting (3) & (4)

$2A = 2x + \delta x$ & $\boxed{B = \delta x/2}$

$A = \frac{2x + \delta x}{2}$

$\boxed{A = x + \delta x/2}$

Comparing (1) & (2)

$\delta y = \cos(\cancel{x+\delta x}) - \cos x$

$= -2\sin(x + \delta x/2)\sin(\delta x/2)$

Dividing through by δx

$\frac{\delta y}{\delta x} = \frac{-2\sin(x + \delta x/2)\sin(\delta x/2)}{\delta x}$

$$\frac{\delta y}{\delta x} = \frac{-\sin\left(x+\delta x/2\right)\sin\left(\delta x/2\right)}{\delta x/2}$$

$$= -\sin\left(x+\delta x/2\right) \times \frac{\sin(\delta x/2)}{\delta x/2}$$

Taking Limit $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\sin \delta x/2}{\delta x/2} = 1$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -\sin\left(x + 0/2\right) \times 1$$

$$\delta y/\delta x = -\sin x$$

8 ~~P = sin t~~ $y = 3t^2$; $x = 1/t^2$ t^{-2}

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 6t \quad ; \quad \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$$

$$= 6t \times \frac{-2}{t^3} = \frac{6 \times -2}{t^2} = \frac{-12}{t^2}$$

$$dy/dx = \frac{-12}{t^2}$$

~~$dy/dx = -12t^{4}$~~

9) $y = x^2 \cos 2x \; e^{4x}$

Solution

Taking $\log_e$ of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiplying both sides by 'y'

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

$$= x^2 \cos 2x \, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$\frac{dy}{du} = \cos u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos 3x^3 + 5$$