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MECHATRONIC ENGINEERING
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① $\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{A(x-1)+B(x+1)}{(x+3)(x-1)}$

$$A(x-1)+B(x+3) = 11-3x$$

let $x = -3$

$$A(-4) = 11+9$$
$$-4A = 20$$
$$A = -5$$

let $x = 1$

$$B(4) = 11-3$$
$$4B = 8$$
$$B = 2$$

$$\int \frac{-5}{x+3}dx + \int \frac{2}{x-1}dx$$

$u = x+3$ $\quad$ $u = x-1$

$\frac{du}{dx} = 1$ $\quad$ $\frac{du}{dx} = 1$

$du = dx$ $\quad$ $du = dx$

$$\int \frac{-5\,du}{u} + \int \frac{2\,du}{u}$$

$$-5\ln u + 2\ln u$$
$$-5\ln(x+3) + 2\ln(x-1)$$

② $\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

$x = -1$

$2 + 9 - 35 = A(-3)(2)$

$-24 = -6A$

$A = 4$

when $x = 2$

$8 - 18 - 35 = B(3)(5)$

$-45 = 15B$

$B = -3$

when $x = 3$

$18 - 27 - 35 = C(4)(2)$

$-44 = 8C$

$C = -\frac{11}{2}$

$$\int \frac{4\,dx}{x+1} - \int \frac{3\,dx}{x-2} + \frac{1}{2}\int \frac{11\,dx}{2(x+3)} = \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx$$

$u = x+1 \qquad u = x-2 \qquad u = 2(x+3)$

$\frac{du}{dx} = 1 \qquad \frac{du}{dx} = 1 \qquad \frac{du}{dx} = 1$

$du = dx \qquad du = dx \qquad du = dx$

$$\int \frac{4\,du}{u} - \int \frac{3\,du}{u} - \frac{1}{2}\int \frac{11}{u}\,du$$

$$4\ln u - 3\ln u - \frac{11}{2}\ln u$$

$$4\ln(x+1) - 3\ln(x-2) - \frac{11}{2}\ln(x+3)$$

(3) $\int \frac{1}{x^2+121} = \int \frac{1}{(x-11)(x+11)}$

$\int \frac{1}{x^2+121} = \int \frac{1}{(x-11)(x+11)}$

$$\int \frac{dx}{(x^2+121)}$$

let $x = 11\tan\theta$

$\frac{dx}{d\theta} = 11\sec^2\theta$

$dx = 11\sec^2\theta\,d\theta$

$x^2 + 11^2 = 11^2 + 11^2\tan^2\theta$

$= 11^2(\tan^2\theta + 1)$

$= 11^2\sec^2\theta$

$$\int \frac{du}{11^2 + u^2}$$

$$\frac{1}{11} \int d\theta$$

$$\frac{1}{11} (\theta)$$

$$\frac{1}{11} \tan^{-1} \frac{u}{11} + C$$