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Assignment

1) $\int \frac{(11-3x)}{x^2+2x-3}\,dx$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$A(x+3) + B(x-1) = 11-3x$

$Ax + 3A + Bx - B = 11-3x$

$A + B = -3 \quad \text{---} \quad (i)$

$3A - B = 11 \quad \text{---} \quad (ii)$

$A = -3 - B \quad \text{---} \quad (iii)$

$3(-3-B) - B = 11$

$-9 - 3B - B = 11$

$-4B = 20$

$B = \frac{-20}{4} = -5$

$A = -3-(-5) = -3+5 = 2$

$\therefore \frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$

$= \int \frac{2}{x-1}\,dx - \int \frac{5}{x+3}\,dx$

Let $u = x-1 \qquad v = x+3$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 1$

$2\int \frac{du}{u} - 5\int \frac{dv}{v}$

$= 2\ln(u) - 5\ln(v) + c$

$= 2\ln(x-1) - 5\ln(x+3) + c$

2 $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$A(x-2)(x+3) + B(x+1)(x-3) + C(x+1)(x-2)$

$= 2x^2 - 9x + 35$

$A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2) = 2x^2-9x-35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C =$

$2x^2 - 9x - 35$

$A + B + C = 2 \quad \ldots (1)$

$A + 4B - C = -9 \quad \ldots (11)$

$-6A + 3B - 2C = -35 \quad \ldots (111)$

Solving ~~by~~ simultaneously; $A = 4, B = -3, C = 1$

$\therefore \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$

$4\int \frac{1}{x+1}\,dx - 3\int \frac{1}{x-2}\,dx + \int \frac{1}{x+3}\,dx$

let $u = x+1$ let $v = x-2$ let $w = x+3$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = 1$ $\frac{dw}{dx} = 1$

$= 4\int \frac{du}{u} - 3\int \frac{dv}{v} + \int \frac{dw}{w}$

$= 4\ln(u) - 3\ln(v) + \ln(w) + c$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$

3) $\int \frac{1}{x^2+121}$

Given a right angled triangle;

$\tan\theta = \frac{x}{11}$

$\therefore x = 11\tan\theta$

$\frac{dx}{d\theta} = 11\sec^2\theta$

$dx = 11\sec^2\theta \cdot d\theta$

Substituting $x = 1\tan\theta$ and $dx = 11\sec^2\theta \cdot d\theta$

$$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2 + 121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta + 121}$$

Recall that; $1+\tan^2\theta = \sec^2\theta$

$\therefore 121\tan^2\theta + 120 = 120\sec^2\theta$

$$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}\left[\theta\right] + c$$

If $\tan\theta = \frac{x}{11}$, $\therefore$ $\theta = \tan^{-1}\left[\frac{x}{11}\right]$

$$= \frac{1}{11}\tan^{-1}\left[\frac{x}{11}\right] + c$$ //