

Adeoye Abdulwasim Omogbolahan

Mechanical Engineering
19/ENGG06/1002

$$1) \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$
$$= \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

$$11-3x = A(x-1) + B(x+3)$$

at $x=1$

$$B(A) = 11-3$$

$$B = 2$$

at $x=3$

$$A(2) = 11-9$$

$$A = 1$$

$$\int \frac{1}{x+3} dx + \int \frac{2}{x-1} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\int \frac{dx}{x+3} + \int \frac{2dx}{x-1} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$= \ln u + 2 \ln u$$

$$= \ln(x+3) + 2 \ln(x-1)$$

$$\int \frac{1}{x^2+121} dx = \int \frac{dx}{x^2+11^2}$$

$$x = 11 \tan \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$\begin{aligned} x^2+11^2 &= 11^2 \tan^2 \theta + 11^2 \\ &= 11^2 (\tan^2 \theta + 1) \\ &= 121 \sec^2 \theta \end{aligned}$$

$$\Rightarrow \int \frac{11 \sec^2 \theta}{121 \sec^2 \theta} d\theta$$

$$= \int \frac{d\theta}{11}$$

$$= \frac{1}{11} \int d\theta$$

$$= \frac{1}{11} [\theta] + C$$

$$= \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$

$$= \frac{\tan^{-1} \frac{x}{11}}{11} + C$$

Substituting $A=4$ in (4)

$$2(A) + 5B = -7$$

$$8 + 5B = -7$$

$$5B = -7 - 8$$

$$= -15$$

$$B = -15 \div 5$$

$$B = -3$$

Substituting $A=4$ and $B=-3$ in (1)

$$A + (-3) + C = 2$$

$$1 + C = 2$$

$$C = 1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$= \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$\begin{aligned} 2x^2 - 9x - 35 &= A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2) \\ &= Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C \\ &= Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C \\ 2x^2 - 9x - 35 &= (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C) \end{aligned}$$

$$A+B+C = 2 \quad \text{--- (1)}$$

$$A+4B-C = -9 \quad \text{--- (2)}$$

$$-6A+3B-2C = -35 \quad \text{--- (3)}$$

$$(1) \quad A+B+C = 2$$

$$(2) \quad A+4B-C = -9$$

$$\text{Add } 2A+5B = -7 \quad \text{--- (4)}$$

$$(3) \quad -6A+3B-2C = -35$$

$$\text{Add } 2A+2B+2C = 4$$