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MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO.: 111

MAT 104 ASSIGNMENT

Integrate the following with respect to their variable

1. $\dfrac{(11-3x)}{x^2+2x-3}$

2. $\dfrac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}$

3. $\dfrac{1}{(x^2+121)}$

Solution

1. $$\int \frac{(11-3x)}{x^2+2x-3}\,dx = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$\therefore A(x+3) + B(x-1) = 11-3x$$

$$Ax + 3A + Bx - B = 11-3x$$

$$Ax + Bx + 3A - B = 11-3x$$

$$(A+B)x + (3A-B) = 11-3x$$

$$A+B = -3$$

$$3A - B = 11$$

$$4A = 8 \Rightarrow A = 2$$

$$B = -3-2 \Rightarrow B = -5$$

Therefore,

$$\int \frac{2\,dx}{x-1} - \int \frac{5\,dx}{x+3}$$

Let $u = x-1$ Let $u = x+3$

$\frac{du}{dx} = 1$ $\frac{du}{dx} = 1$

$du = dx$ $du = dx$

$\therefore 2\int \frac{du}{u}$ $5\int \frac{du}{u}$

$= 2\ln u$ $= 5\ln u$

$= 2\ln(x-1)$ $= 5\ln(x+3)$

$\Rightarrow 2\ln(x-1) - 5\ln(x+3) + C$

2. $\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}\,dx$

$\Rightarrow \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$

~~At~~ Multiply through by $(x+1)(x-2)(x+3)$

$\Rightarrow A(x^2+3x-2x-6) + B(x^2+3x+x+3) + C(x^2-2x+x-2) =$
$2x^2-9x-35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$

$(A+B+C)x^2 + (A+4B-C)x - 6A + 3B - 2C = 2x^2 - 9x - 35$

$A + B + C = 2 \longrightarrow ①$

$A + 4B - C = -9 \longrightarrow ②$

$-6A + 3B - 2C = -35$ —③

From ①,

$A = 2 - B - C$ —④

Substitute ④ into ②

$2 - B - C + 4B - C = -9$

$3B - 2C = -11$ —⑤

Substitute ⑤ into ③

$-6A - 11 = -35$

$6A = 35 - 11$

$6A = 24$

$A = 4$

$\therefore 4 = 2 - B - C$

$2 = -B - C$

$B = -C - 2$ —⑥

Substitute ⑥ into ⑤

$\therefore 3(-C-2) - 2C = -11$

$-3C - 6 - 2C = -11$

$-5C = -5$

$C = 1$

$\therefore B = -1 - 2$

$= -3$

Therefor,

$$\int \frac{4\,dx}{x+1} = \int \frac{3\,dx}{x-2} + \int \frac{dx}{x+5}$$

$u = x+1$ $\quad$ $u = x-2$ $\quad$ $u = x+3$

$du = dx$ $\quad$ $du = dx$ $\quad$ $du = dx$

$\therefore 4\int \frac{du}{u}$ $\quad$ $3\int \frac{du}{u}$ $\quad$ $\int \frac{du}{u}$

$= 4\ln u$ $\quad$ $= 3\ln u$ $\quad$ $= \ln u$

$= 4\ln(x+1)$ $\quad$ $= 3\ln(x-2)$ $\quad$ $= \ln(x+3)$

$$\Rightarrow 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$

3. $$\int \frac{1}{(x^2+121)}\,dx = \int \frac{dx}{(x^2+11^2)}$$

$x = 11\tan\theta$

$\frac{dx}{d\theta} = 11\sec^2\theta$

$dx = 11\sec^2\theta\, d\theta$

Also,

$x^2 + 11^2 = 11^2\tan^2\theta + 11^2$

$= 11^2(\tan^2\theta + 1)$

$= 121\sec^2\theta$

$$\therefore \int \frac{dx}{(x^2+11^2)} = \int \frac{11\cancel{\sec^2\theta}\,d\theta}{121\cancel{\sec^2\theta}}$$

$$= \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}[\theta] + C$$

$$= \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$

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