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Mechanical Engineering

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MAF104

$$1. \int \frac{(11-3x)}{x^2+2x-3} dx$$

$$\int \frac{11-3x}{x^2+3x-x-3}$$

$$\int \frac{11-3x}{x(x+3)-1(x+3)}$$

$$\int \frac{11-3x}{(x+3)(x-1)}$$

$$\frac{A}{x+3} + \frac{B}{x-1} = \frac{11-3x}{(x+3)(x-1)}$$

$$A(x-1) + B(x+3) = 11-3x$$

When $x = -3$,

$$A(-3-1) + B(-3+3) = 11-3(-3)$$

$$-4A = 11+9$$

$$\frac{-4A}{-4} = \frac{20}{-4}$$

$$A = -5$$

When $x = 1$,

$$A(1-1) + B(1+3) = 11-3(1)$$

$$\frac{4B}{4} = \frac{8}{4}$$

$$B = 2$$

$$\frac{-5}{x+3} + \frac{2}{x-1}$$

$$\int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$$2 \ln(x-1) - 5 \ln(x+3) + C$$

$$\int \frac{(11-3x)}{x^2+2x-3} dx = 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2. \int \frac{6x^2 - 9x - 55}{(x+1)(x-2)(x+3)} dx$$

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} = \frac{2x^2 - 9x - 55}{(x+1)(x-2)(x+3)}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2 - 9x - 55$$

$$A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2) = 2x^2 - 9x - 55$$

$$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 55$$

$$x^2(A+B+C) + x(A+4B-C) - 6A+3B-2C = 2x^2 - 9x - 55$$

$$A+B+C = 2 \dots i$$

$$A+4B-C = -9 \dots ii$$

$$-6A+3B-2C = -35 \dots iii$$

$$\text{from (i)} \quad A = 2 - B - C$$

$$2 - B - C + 4B + C = -9$$

$$-6(2 - B - C) + 3B - 2C = -35$$

$$-12 + 6B + 6C + 3B - 2C = -35$$

$$9B + 4C = -35 + 12$$

$$9B + 4C = -23$$

$$2 - B - C + 4B - C = -9$$

$$3B - 2C = -9 - 2$$

$$3B - 2C = -11 \dots iv$$

$$\text{from (ii)} \quad -6A - 11 = -35$$

$$-6A = -35 + 11$$

$$-6A = -24 \quad \therefore A = 4$$

$$\text{from (i)} \quad 4 + B + C = 2$$

$$B + C = 2 - 4 = -2$$

$$B = -2 - C$$

$$\text{from (iv)} \quad 3(-2 - C) - 2C = -11$$

$$-6 - 3C - 2C = -11$$

$$-5C = -5 \quad C = 1$$

$$\text{from (i)} \quad 4 + B + 1 = 2$$

$$B = 2 - 5 = -3$$

$$\int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$

$$3. \int \frac{1}{(x^2+121)} dx$$

$$\int \frac{1}{x^2+11^2} dx$$

$$x = 11 \tan \theta$$

$$\frac{dx}{d\theta} = 11 \sec^2 \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2 + 11^2 = \cancel{x^2} + 11^2 \tan^2 \theta + 11^2$$

$$= 11^2 (\tan^2 \theta + 1)$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$x^2 + 11^2 = 11^2 \sec^2 \theta$$

$$\int \frac{11 \sec^2 \theta d\theta}{11^2 \sec^2 \theta}$$

$$\int \frac{1}{11} d\theta$$

$$\frac{1}{11} \int d\theta$$

$$= \frac{1}{11} \theta + C$$

$$\frac{x}{11} = \frac{11 \tan \theta}{11}$$

$$\frac{x}{11} = \tan \theta$$

$$\tan^{-1}\left(\frac{x}{11}\right) = \theta$$

$$\therefore \frac{1}{11} \tan^{-1}\left(\frac{x}{11}\right) + C$$

$$\therefore \int \frac{1}{(x^2+121)} dx = \frac{1}{11} \tan^{-1}\left(\frac{x}{11}\right) + C$$