Serial No:- 096

ISAAC EXB GRACE

19/ENG08/004

1) $\int \frac{11-3x}{x^2+2x-3} dx$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$A(x+3) + B(x-1) = 11-3x$

$Ax + 3A + Bx - B = 11-3x$

$A + B = -3$ — (1)

$3A - B = 11$ — (2)

$A = -3 - B$ — (iii) $\quad 3(-3-B) - B = 11$

$-9 - 3B - B = 11$

$-4B = 20$

$B = -20/4 = -5$

$A = -3 - (-5) = -3 + 5 = 2$

$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$

$= \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$

let $u = x-1 \qquad r = x+3$

$du/dx = 1 \qquad dr/dx = 1$

$2\int \frac{du}{u} - 5\int \frac{dr}{r}$

$= 2\ln(u) - 5\ln(r) + C$

$= 2\ln(x-1) - 5\ln(x+3) + C$

(2) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2-9x-35$

$A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2) = 2x^2-9x-35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2-9x-35$

$A + B + C = 2$ — (1)

$A + 4B - C = -9$ — (2)

$-6A + 3B - 2C = -35$ — (3)

Simultaneously :- $A = 4, B = -3, C = 1$

$$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$4\int \frac{1}{x+1}\,dx - 3\int \frac{1}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

let $u = x+1$ let $v = x-2$ let $w = x+3$

$du/dx = 1$ $dv/dx = 1$ $dw/dx = 1$

$$= 4\int \frac{du}{u} - 3\int \frac{dv}{v} + \int \frac{dw}{w}$$

$4\ln(u) - 3\ln(v) + \ln(w) + c$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$

3) $$\int \frac{1}{x^2+121}\,dx$$

Given a right angled triangle $\tan\theta = x/11$

$\therefore x = 11\tan\theta$ $dx/d\theta = 11\sec^2\theta$

$dx = 11\sec^2\theta \cdot d\theta$

Substituting $x = 11\tan\theta$ and $dx = 11\sec^2\theta \cdot d\theta$

$$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2 + 121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta + 121}$$

Recall that: $1 + \tan^2\theta = \sec^2\theta$

Recal

$\therefore 121\tan^2\theta + 121 = 121\sec^2\theta$

$$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}[\theta] + c$$

If $\tan\theta = \frac{x}{11}$ $\therefore \theta = \tan^{-1}[x/11]$

$$= \frac{1}{11}\tan^{-1}\left[\frac{x}{11}\right] + c$$