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Bio Medical Engineering
19/engor/005

1) $\int \frac{11-3x}{x^2+2x-3} dx$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$A(x+3) + B(x-1) = 11-3x$

$Ax + 3A + Bx - B = 11-3x$

$A+B = 3$ —① $3A-B = 11$ —②

$A = -3-B$ —③ $3(-3-B) - B = 11$ $-9-3B-B=11$

$-4B = 20$ $B = -20/4 = -5$

$A = -(-5) = -3+5 = 2$

$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$

$= \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$

Let $u = x-1$ $v = x+3$

$du/dx = 1$ $dv/dx = 1$

$2\int \frac{du}{u} - 5\int \frac{dv}{v}$

$= 2\ln(u) - 5\ln(v) + c$

$= 2\ln(x-1) - 5\ln(x+3) + c$

$$\int \frac{1}{x^2+121}\,dx$$

Given a right angle triangle two

$\tan\theta = x/11$

$x = 11\tan\theta$ $\quad dx/d\theta = 11\sec^2\theta$

$dx = 11\sec^2\theta \cdot d\theta$

Substituting $x = 11\tan\theta$ and $dx = 11\sec^2\theta\, d\theta$

$$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2+121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta + 121}$$

Recall that $1 + \tan^2\theta = \sec^2\theta$

$\therefore 121\tan^2\theta + 121 = 121\sec^2\theta$

$$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}[\theta] + c$$

$= 1/11 \tan\theta = x/11 \quad \theta = \tan^{-1}[x/11]$

$= 1/11 \tan^{-1}[x/11] + c$

2) $\int \frac{2x^2 - 9x - 35}{x(x+1)(x-2)(x+3)}\,dx$

$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2 - 9x - 35$

$A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2) = 2x^2 - 9x - 35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$

$A + B + C = 2 \quad -①$

$A + 4B - C = -9 \quad -②$

$-6A + 3B - 2C = -35 \quad -③$

Solving Simultaneously

$A = 4, B = -3, C = 1$

$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$

$4\int \frac{1}{x+1}\,dx - 3\int \frac{1}{x-2}\,dx + \int \frac{1}{x+3}\,dx$

Let $u = x+1$ $\quad$ Let $v = x-2$ $\quad$ Let $w = x+3$

$du/dx = 1$ $\quad$ $dv/dx = 1$ $\quad$ $dw/dx = 1$

$= 4\int \frac{du}{u} - 3\int \frac{dv}{v} + \int \frac{dw}{w}$

$= 4\ln(u) - 3\ln(v) + \ln(w) + c$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$