

$$A + B + C = 2 \quad \dots (i)$$

$$A + 4B - C = -9 \quad \dots (ii)$$

$$A - 6A + 3B - 2C = -35 \quad \dots (iii)$$

$$i) \quad A + B + C = 2$$

$$ii) \quad A + 4B - C = -9$$

$$\text{And } 2A + 5B = -7 \quad \dots (iv)$$

$$iii) \quad -6A + 3B - 2C = -35$$

$$\frac{2A + 5B + 2C = 4}{-4A + 5B = -31} \quad \dots (v)$$

$$\text{And } -4A + 5B = -31 \quad \dots (v)$$

$$ii) \quad 2A + 5B = -7$$

$$v) \quad -4A + 5B = -31$$

$$6A = 24$$

$$A = 24/6$$

$$A = 4$$

Substituting $A = 4$ in (iv)

$$2(4) + 5B = 7$$

$$8 + 5B = 7$$

$$5B = 7 - 8$$

$$5B = -1$$

$$B = -1/5$$

Substituting $A = 4$ and $B = -1/5$ in (i)

$$4 + (-1/5) + C = 2$$

$$1 + C = 2$$

$$C = 1$$

$$\int \frac{2x^2 - 9x - 55}{(x-1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$

$$3.) \int \frac{1}{x^2+121} dx = \int \frac{dx}{x^2+11^2}$$

$$x = 11 \tan \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2+11^2 = 11^2 \tan^2 \theta + 11^2$$

$$= 11^2 (\tan^2 \theta + 1)$$

$$= 121 \sec^2 \theta$$

$$\int \frac{11 \sec^2 \theta d\theta}{121 \sec^2 \theta}$$

$$= \int \frac{d\theta}{11}$$

$$= \frac{1}{11} \int d\theta$$

$$= \frac{1}{11} (\theta) + C$$

$$= \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$

$$= \frac{\tan^{-1} x/11}{11} + C$$

Integrate the following with respect to the variable

$$1) \int \frac{(11-3x)}{x^2+2x-3} dx = \frac{11-3x}{x^2+2x-3} = \frac{A(11-3x)}{(x+3)(x-1)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$11-3x = A(x-1) + B(x+3)$$

$$\text{at } x = 1$$

$$B(4) = 11-3$$

$$B = 2$$

$$\text{at } x = 3$$

$$A(2) = 11-9$$

$$A = 1$$

$$\begin{aligned} &= \int \frac{1}{(x+3)} dx + \int \frac{2}{(x-1)} dx = \int \frac{11-3x}{x^2+2x-3} dx \\ &= \int \frac{dx}{x+3} + \int \frac{2dx}{x-1} = \int \frac{11-3x}{x^2+2x-3} dx \\ &= \ln|x+3| + 2\ln|x-1| \end{aligned}$$

$$2) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$= \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$= Ax^2 + Bx^2 + Cx^2 + Ax - 6A + Bx + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$