

19/ENG06/007

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Mechanical Engineering

MAE 104

Integrate the following with respect to their variable

1.  $(11-3x)/x^2+2x-3$

Sol<sup>n</sup>

$$\int \frac{11-3x}{x^2+2x-3}$$

$$\int \frac{11-3x}{x^2+3x-x-3}$$

$$\int \frac{11-3x}{x(x+3)+-(x+3)}$$

$$\int \frac{11-3x}{(x+3)(x-1)}$$

$$\frac{A}{x+3} + \frac{B}{x-1} = \frac{11-3x}{(x+3)(x-1)}$$

$$A(x-1) + B(x+3) = 11-3x$$

$$A(-3-1) = 11-3(-3) \text{ where } x = -3$$

$$-4A = 20$$

$$A = -5$$

$$\text{where } x = 1$$

$$B(1+3) = 11 - 3(1)$$

$$4B = 8$$

$$B = 2$$

$$\frac{-5}{x+3} + \frac{2}{x-1}$$

$$\int \frac{2dx}{x-1} - \int \frac{5dx}{x+3}$$

$$2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2 \frac{(2x^2 - 9x - 35)}{(x+1)(x-2)(x+3)}$$

Solve

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$(x+1)(x-2)(x+3)$$

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} = \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$= 2x^2 - 9x - 35$$

$$A(x^2 + 3x - 2x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2) = 2x^2 - 9x - 35$$

$$\cancel{Ax^2 + 3Ax - 2Ax - 6A} + B(x^2 + 4x + 3) + C(x^2 - x - 2) = 2x^2 - 9x - 35$$

$$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$$

$$= 2x^2 - 9x - 35$$

$$(A+B+C)x^2 + (A+4B-C)x - 6A+3B - 2C = 2x^2 - 9x - 35$$

$$A+B+C = 2$$

$$A+4B-C = 2$$

$$-6A+3B-2C = -35$$

$$A = 2 - B - C$$

$$2 - B - C + 4B - C = -9$$

$$3B - 2C = -11 \quad \text{--- (1)}$$

$$-6A - 11 = -35$$

$$6A = 35 - 11$$

$$6A = 24$$

$$A = 4$$

$$\begin{array}{l} 2 \cancel{A} - B - C \\ B = -C \end{array}$$

$$\text{Recall } 2 = -B - C$$

$$B = -C - 2$$

$$3(-C - 2) - 2C = -11$$

$$-3C - 6 - 2C = -11$$

$$-3C - 2C = -11 + 6$$

$$-5C = -5$$



$$C = 1$$

$$B = -1 - 2$$

$$B = -3$$

$$\int \frac{4dx}{x+1} + \int \frac{-3dx}{x-2} + \int \frac{dx}{x+3}$$

$$4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + C$$

$$\textcircled{3} \frac{1}{x^2+121}$$

$$\int \frac{dx}{x^2+121}$$

$$\int \frac{dx}{x^2+11^2}$$

$$x = 11 \tan \theta$$

$$\frac{dx}{d\theta} = 11 \sec^2 \theta$$

$$d\theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2 + 11^2 = 11^2 \tan^2 \theta + 11^2$$

$$= 11^2 (\tan^2 \theta + 1)$$

$$11^2 \sec^2 \theta + 11^2 = 11^2 \sec^2 \theta$$

$$x^2 + 11^2 = 11^2 \sec^2 \theta$$

$$\frac{\int 11 \sec^2 \theta d\theta}{11^2 \sec^2 \theta}$$

$$\frac{1}{11} \int d\theta$$

$$\frac{1}{11} \theta + C$$

$$x = 11 \tan \theta$$

$$\frac{x}{11} = \tan \theta$$

11

$$\tan^{-1} \left( \frac{x}{11} \right) = \theta$$

$$\frac{1}{11} \tan^{-1} \left( \frac{x}{11} \right) + C$$

11