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Electrical Electronic Engg

MAT 104 Serial No: 121

19/ENG04/040

$$\int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$A(x+3) + B(x-1) = 11-3x$$

$$Ax + 3A + Bx - B = 11-3x$$

$$A + B = -3 \rightarrow (1) \quad 3A - B = 11 \rightarrow (2)$$

$$A = -3 - B \rightarrow (3) \quad 3(-3 - B) - B = 11 \quad -9 - 3B - B = 11$$

$$-4B = 20 \quad B = -20/4 = -5$$

$$A = -3 - (-5) = -3 + 5 = 2$$

$$\therefore \frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$= \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$$\text{Let } u = x-1 \quad v = x+3$$

$$\frac{du}{dx} = 1$$

$$\frac{dv}{dx} = 1$$

$$2 \int \frac{du}{u} - 5 \int \frac{dv}{v}$$

$$= 2 \ln(u) - 5 \ln(v) + C$$

$$= 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2) = 2x^2 - 9x - 35$$

$$Ax^2 + Bx + C = 2x^2 - 9x - 35$$

$$A + B + C = 2 \rightarrow (1)$$

$$A + 4B - C = -9 \rightarrow (2)$$

$$-6A + 3B - 2C = -35 \rightarrow (3)$$

Solving simultaneously, $A = 4, B = -3, C = 1$

$$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$+ \int \frac{1}{x+1} dx - 3 \int \frac{1}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\text{Let } u = x+1 \quad \text{Let } v = x-2 \quad \text{Let } w = x+3$$

$$\frac{du}{dx} = 1$$

$$\frac{dv}{dx} = 1$$

$$\frac{dw}{dx} = 1$$

$$= 4 \int \frac{du}{u} - 3 \int \frac{dv}{v} + \int \frac{dw}{w}$$

$$= 4 \ln(u) - 3 \ln(v) + \ln(w) + C$$

$$4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + C$$

$$\int \frac{1}{x^2 + 121} dx$$

Given a right angled triangle, $\tan \theta = x/11$

$$\therefore \text{Let } x = 11 \tan \theta \quad \frac{dx}{d\theta} = 11 \sec^2 \theta \quad dx = 11 \sec^2 \theta \cdot d\theta$$

$$\text{Substituting } x = 11 \tan \theta \text{ and } dx = 11 \sec^2 \theta \cdot d\theta$$

$$\int \frac{11 \sec^2 \theta \cdot d\theta}{(11 \tan \theta)^2 + 121} = \int \frac{11 \sec^2 \theta \cdot d\theta}{121 \tan^2 \theta + 121}$$

$$\text{Recall that, } 1 + \tan^2 \theta = \sec^2 \theta$$

$$121 \tan^2 \theta + 121 = 121 \sec^2 \theta$$

$$= \int \frac{11 \sec^2 \theta \cdot d\theta}{121 \sec^2 \theta} = \int \frac{d\theta}{11} = \frac{1}{11} \int d\theta$$

$$= \frac{1}{11} [\theta] + C$$

$$= \frac{1}{11} \tan^{-1} \left[\frac{x}{11} \right] + C$$

$$= \frac{1}{11} \tan^{-1} \left[\frac{x}{11} \right] + C$$