Assignment: MAT104 Practice Questions. Course Code: MAT104
Course Title: General Mathematics III. lecturer: Dr Ojelami.
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Questions.

1. Integrate the following with respect to their variable

(i) $(11-3x)/x^2+2x-3$

Solution.

$$\int\left(\frac{11-3x}{x^2+2x-3}\right)dx \quad , \text{ Integrating by partial fraction}$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x^2+3x)(-x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

Simplifying $= \dfrac{A(x+3)+B(x-1)}{(x-1)(x+3)} = \dfrac{11-3x}{(x^2+3x)(-x-3)}$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

multiplying both sides by $(x-1)(x+3)$

$$\cancel{(x-1)(x+3)} \times \frac{11-3x}{\cancel{(x-1)(x+3)}} = \frac{A(x+3)+B(x-1)}{\cancel{(x-1)(x+3)}} \times \cancel{(x-1)(x+3)}$$

$\therefore$ $11-3x = A(x+3) + B(x-1)$

at $x+3=0$, $x=-3$, $B(x-1) = 11-3x$

$$B(-3-1) = 11-3(-3)$$
$$B(-4) = 11+9$$
$$B(-4) = 20$$
$$B = 20/-4 = -5$$

At $x-1=0$, $x=1$, $A(x+3) = 11-3x$

$$A(1+3) = 11-3(1)$$
$$A(4) = 8$$
$$A = 2$$

$$\Rightarrow \int\frac{2}{x-1}dx + \int\frac{-5}{x+3}dx = \int\frac{A}{x-1}dx + \int\frac{B}{x+3}dx$$

$$= \int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}\,dx$$

let $u = x-1$ let $u = x+3$

$\frac{du}{dx} = 1$ $\frac{du}{dx} = 1$, $du = dx$

$\therefore du = dx$ $-5\int \frac{dx}{x+3}$

$= 2\int \frac{dx}{x-1}$ $-5\int \frac{du}{u}$

$= 2\int \frac{du}{u}$

$= 2\ln u$ $= -5\ln u.$

$= 2\ln u - 5\ln u$

$$\therefore \int\left(\frac{11-3x}{(x^2+2x-3)}\right)dx = 2\ln u - 5\ln u$$

$$= 2\ln(x-1) - 5\ln(x+3) + C$$

ans.

2. $2x^2-9x-35 \,/\, (x+1)(x-2)(x+3)$

Solution

$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$ by partial fraction

$$= \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} = \frac{A(x-2)(x-3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$\therefore 2x^2-9x-35 = (2x^2-14x)(+5x-35)$

$= 2x(x-7)+5(x-7)$

$= (2x+5)(x-7)$

Note:-

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x-3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

multiplying both sides by $(x+1)(x-2)(x+3)$, we have:-

$2x^2-9x-35 = A(x^2+3x-2x-6)+B(x^2+3x+x+3)+C(x^2-x-2)$

$2x^2-9x-35 = A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C$

$2x^2-9x-35 = x^2(A+B+C)+x(A+4B-C)+(-6A+3B-2C)$ ---(ii)

Comparing (ii) to $2x^2 - 9x - 35$ —

$\therefore$ $A + B + C = 2$, $A + 4B - C = -9$ - - - (iii)

$-6A + 3B - 2C = 0$ - - - (iv)

from $A + B + C = 2$

$A = 2 - B - C$

Subs $2 - B - C$ for A in eqn (iii), $(2 - B - C) + 4B - C = -9$

$= 2 - B - C + 4B - C = -9$

$2 + 3B - 2C = -9$, $3B - 2C = -9 - 2$

$3B - 2C = -11$ - - - (v)

from (iv), $-6A + 3B - 2C = 0$

$= -6(2 - B - C) + 3B - 2C = 0$

$= -12 + 6B + 6C + 3B - 2C = 0$

$= -12 + 9B + 6C - 2C = 0$

$\therefore 9B + 4C = 12$ - - - (vi)

from (v) $3B - 2C = -11$, $\frac{3B}{3} = \frac{-11 + 2C}{3}$

$B = \frac{-11 + 2C}{3}$ - - - (vii)

Put (vii) in (vi), $\overset{3}{\cancel{9}}\left(\frac{-11 + 2C}{\cancel{3}}\right) + 4C = 12$

$3(-11 + 2C) + 4C = 12$

$-33 + 6C + 4C = 12$

$-33 + 10C = 12$

$10C = 12 + 33$

$10C = 45$, $C = \frac{45}{10} = 9/2$ ans

Subs $9/2$ for C in eqn (v)

$3B - 2C = -11$

$3B - \cancel{2}(9/\cancel{2}) = -11$

$3B - 9 = -11$, $3B = -11 + 9$

$3B = -2$, $B = \frac{-2}{3}$ // ans

but $A + B + C = 2$

$\therefore$ $A = -B - C = 2 - \left(\frac{-2}{3}\right) - 9/2$

$A = \frac{2}{1} + \frac{2}{3} - \frac{9}{2} = \frac{12+4-27}{6} = \frac{-11}{6}$

$$\Rightarrow \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \int \frac{-11/6\,dx}{(x+1)} + \int \frac{+2/3\,dx}{(x-2)} + \int \frac{9/2}{x+3}\,dx$$

$= \frac{-11}{6}\int \frac{dx}{x+1}$, let $u = x+1$, $\frac{du}{dx} = 1$, $du = dx$

$\therefore -\frac{11}{6}\int \frac{du}{u} = -\frac{11}{6}\ln u$

$= -\frac{2}{3}\int \frac{dx}{x-2}$, let $u = x-2$, $\frac{du}{dx} = 1$, $du = dx$

$\therefore -\frac{2}{3}\int \frac{du}{u} = -\frac{2}{3}\ln u.$

$= \frac{9}{2}\int \frac{dx}{x+3}$, let $u = x+3$, $\frac{du}{dx} = 1$, $du = dx$

$\frac{9}{2}\int \frac{du}{u} = \frac{9}{2}\ln u.$

$$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{-11}{6}\ln(x+1) - \frac{2}{3}\ln(x-2) + \frac{9}{2}\ln(x+3 + C$$

ans

3) $\frac{1}{(x^2+121)}$

solution

$$\int \frac{1}{x^2+121}\,dx = \int \frac{1}{x^2+11^2}\,dx$$

$x = 11\tan\theta$

$\frac{dx}{d\theta} = 11\sec^2\theta$, $dx = 11\sec^2\theta\,d\theta$

$x^2+11^2 = ll\tan^2\theta + 11^2 = 11^2(\tan^2\theta + 1)$

factoring, after substituting

$$\int \frac{11\sec^2\theta\,d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta = \frac{1}{11}(\theta) + C$$

but $\theta = \tan^{-1}\frac{x}{11}$

$\therefore \frac{1}{11}\tan^{-1}\frac{x}{11} + C$

ans.