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MECHATRONICS ENGINEERING

MAT 104 PRACTICE QUESTIONS

~~Physical~~

$$(1) \int \frac{11-3x}{x^2+2x-3} dx \quad \therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x-1)(x+3)} \cdot \frac{11+3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\begin{aligned} x^2+2x-3 \\ x^2-x+3x-3 \\ x(x-1)+3(x-1) \\ \therefore (x-1)(x+3) \end{aligned}$$

$$\therefore \frac{11-3x}{(x+1)(x+3)} = \frac{A(x+3)}{(x+1)(x+3)} + \frac{B(x-1)}{(x+1)(x+3)}$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x+3)(x-1)} dx = \frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$\therefore 11-3x = \frac{A(x-1)}{(x+3)(x-1)} + \frac{B(x+3)}{(x+3)(x-1)}$$

$$\frac{(x+3)(x-1)}{(x+3)(x-1)} \quad \frac{(x+3)(x-1)}{(x+3)(x-1)}$$

$$\text{At } x=1$$

$$\therefore 11-3(1) = B(1+3)$$

$$4B = 8$$

$$B = 8/4 = 2$$

$$\text{At } x=-3$$

$$11-3(-3) = A(-3-1)$$

$$20 = -4A$$

$$A = 20/-4 = -5$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{-5}{x+3} dx + \int \frac{2}{x-1} dx$$

$$\text{let } u=x+3$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\therefore \int \frac{-5 du}{u}$$

$$= -5 \ln u du$$

$$\text{let } u=x-1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\therefore \int \frac{2 du}{u}$$

$$= 2 \ln u du$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} = -5 \ln(x+3) + 2 \ln(x-1) + C_1$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3) + C_1$$

$$(2) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\therefore \frac{(x-7)(x+2.5)}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\therefore \frac{(x-7)(x+2.5)}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

when $x = 2$

$$(2-7)(2+2.5) = B(2+1)(2+3)$$

$$(-5 \times 4.5) = B(3 \times 5)$$

$$-22.5 = 15B$$

$$B = -1.5 = -\frac{3}{2}$$

when $x = -1$

$$\therefore (-1-7)(-1+2.5) = A(-1-2)(-1+3)$$

$$(-8 \times 1.5) = A(-3 \times 2)$$

$$-12 = -6A$$

$$A = \frac{12}{6} = 2$$

when $x = -3$

$$\therefore (-3-7)(-3+2.5) = C(-3+1)(-3-2)$$

$$(-10 \times -0.5) = C(-2 \times -5)$$

$$5 = 10C$$

$$C = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{2}{x+1} dx + \int \frac{-1.5}{x-2} dx + \int \frac{0.5}{x+3} dx$$

$$\text{let } u = x+1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$du = dx$$

$$\therefore \int \frac{2}{u} du$$

$$= 2 \ln u + C_1$$

$$\text{let } u = x-2$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\therefore \int \frac{-1.5}{u} du$$

$$= -1.5 \ln u + C_2$$

$$\text{let } u = x+3$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\therefore \int \frac{0.5}{u} du$$

$$= 0.5 \ln u + C_3$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 2 \ln(x+1) - \frac{3}{2} \ln(x-2) + \frac{1}{2} \ln(x+3) + C$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 2 \ln(x+1) + 0.5 \ln(x+3) - 1.5 \ln(x-2) + C$$

$$(3) \int \frac{1}{x^2+121} dx$$

$$\text{Let } x = a \tan \theta$$

from $x^2 + 121$ comparing with $x^2 + a^2$

$$a^2 = 121$$

$$\therefore a = 11$$

$$\therefore x = 11 \tan \theta$$

$$\therefore \frac{dx}{d\theta} = 11 \sec^2 \theta$$

$$\therefore dx = 11 \sec^2 \theta d\theta$$

$$\therefore x^2 + 121 = (11 \tan \theta)^2 + 121$$

$$= 121 \tan^2 \theta + 121$$

$$x^2 + 121 = 121 (\tan^2 \theta + 1)$$

Recall that $\tan^2 \theta + 1 = \sec^2 \theta$

$$\therefore x^2 + 121 = 121 \sec^2 \theta$$

$$\therefore \int \frac{1}{x^2+121} dx = \int \frac{1}{121 \sec^2 \theta} 11 \sec^2 \theta d\theta$$

$$= \int \frac{11 \sec^2 \theta d\theta}{121 \sec^2 \theta}$$

$$= \frac{1}{11} \int d\theta$$

$$= \frac{1}{11} [\theta] + C$$

$$= \frac{1}{11} [\theta] + C$$

$$\therefore \int \frac{1}{x^2+121} dx = \frac{1}{11} \tan^{-1} \left(\frac{x}{11} \right) + C$$