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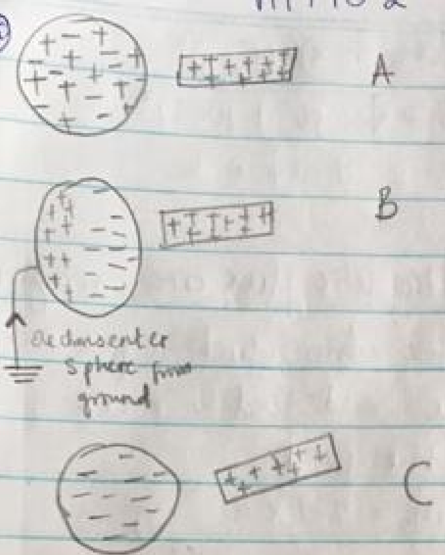
MECHATRONICS ENGINEERING
19/ENG05/006

Numbers Attempted
1, 2, 4, 5

9/4/20

PHY102

1) (a)



(b) $F = \frac{kq_1q_2}{r^2}$

$1 \times (2)^2 = 9\times10^9 q_1q_2$

$4/9\times10^9 = q_1q_2$

$4.44\times10^{-10}c = q_1q_2$ —— i

$5\times10^{-5}\ c = q_1 + q_2$ —— ii

from equ (i) $q_1 = 4.44\times10^{-10}/q_2$ —— iii

insert (iii) into (ii)

$5\times10^{-5}\ c = \frac{4.44\times10^{-10}}{q_2} + q_2$

$5\times10^{-5} = \frac{4.44\times10^{-10} + q_2^2}{q_2}$

$5\times10^{-5} q_2 = 4.44\times10^{-10} + q_2^2$

$q_2^2 - 5\times10^{-5} q_2 + 4.44\times10^{-10} = 0$

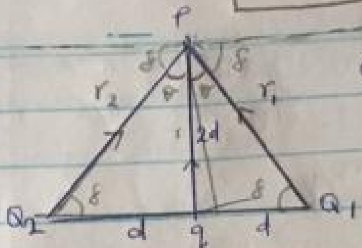
let $a = 1$, $b = -5\times10^{-5}$, $c = 4.44\times10^{-10}$

$q_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$q_2 = 3.844\times10^{-5}c$ or $1.16\times10^{-5}C$

$q_1 = 5\times10^{-5} - q_2$

$\therefore q_1 = 1.16\times10^{-5}C$ or $3.84\times10^{-5}C$



$Q_1 = Q_2 = 8\mu C = 8\times10^{-6}C$

$d = 0.5m$

$E_1 = \frac{kq_1}{r_1^2}$

$E_1 = \frac{9\times10^9 \times 8\times10^{-6}}{1.25}$

$E_1 = 57600 N/C$

Using pythagoras
$r_1^2 = (2d)^2 + d^2$
$r_1^2 = 1^2 + 0.5^2$
$r_1 = \sqrt{1+0.25} = \frac{\sqrt{5}}{2}$
$r_1 = r_2$
$\hat{Q_1} = \hat{Q_2} = \sin^{-1}(1/\frac{\sqrt{5}}{2}) = 63.43$
~~$\theta = 90 - 63.43 = 26.57$~~

$E_2 = \frac{kq_2}{r_2^2} = \frac{9\times10^9\times8\times10^{-6}}{1.25} = 57600N/C$

$E_3 \text{ at } q = 9\times10^9 q\ N/C$

Vectors (N/C)	x-component	y-component
$E_1 = 57600$	$57600\cos 63.43 = 25764$	$-57600\sin 63.43 = -51516.8$
$E_2 = 57600$	$-57600\cos 63.43 = -25764$	$-57600\sin 63.43 = -51516.8$
$E_3 = 9\times10^9 q$	$9\times10^9 q \cos 90° = 0$	$-9\times10^9 q \sin 90° = -9\times10^9 q$
	Σx-component $= 0$	Σy-component $= -103033.6 - 9\times10^9 q$

$E = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$0 = \sqrt{0 + (-103033.6 - 9\times10^9 q)^2}$

$0 = -103033.6 - 9\times10^9 q$

$\frac{103033.6}{-9\times10^9} = q$

$q = -1.14\times10^{-5}C$

(2)(a) Electric field $\vec{E}$ is a region around a charge in which it exerts electrostatic force on another charge while the electric field intensity is the strength of electric field at any point in space, $E = kq/r^2$

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(2b) $Q_1 = 8nC = 8 \times 10^{-9} C$

$Q_2 = 12 \times 10^{-9} C$

$E = Kq/r^2$

(i)

$$E_1 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.469 \, N/C$$

$$E_2 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \, N/C$$

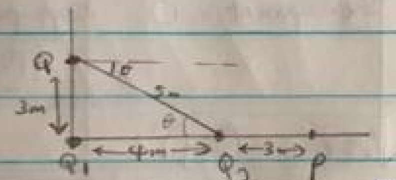
vectors (N/C)	x-component	y component
$E_1 = 1.469$	$1.469 \cos 0°$ $= 1.469$	$1.469 \sin 0°$ $= 0$
$E_2 = 12$	$12 \cos 0°$ $= 12$	$12 \sin 0°$ $= 0$
	$\Sigma x = 13.469$	$\Sigma y = 0$

$E = \sqrt{(\Sigma f x)^2 + (\Sigma f y)^2}$

$E = \sqrt{13.469^2 + 0^2}$

$E = 13.469 \, N/C$

(ii)



Using Pythagoras, $r_2 = \sqrt{3^2 + 4^2}$

$r_2 = 5m$

Using trigonometry, $\theta = \sin^{-1}(3/5)$

$\theta = 36.87°$

$$E_1 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \, N/C$$

$$E_2 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \, N/C$$

vectors (N/C)	x-component	y-component
$E_1 = 8$	$8 \cos 90°$ $= 0$	$+8 \sin 90°$ $= +8$
$E_2 = 4.32$	$4.32 \cos 36.87$ $= 3.46$	$+4.32 \sin 36.87$ $= +2.59$
	$\Sigma x = 3.46$	$\Sigma y = +10.59$

$E = \sqrt{(\Sigma f x)^2 + (\Sigma f y)^2}$

$E = \sqrt{3.46^2 + 10.59^2}$

$E = 11.14 \, N/C$

(4a) Magnetic flux are imaginary lines along which a free north pole would tend to move if placed in the field.

(4b) $m_e = 9.11 \times 10^{-31} kg$

$r = 1.4 \times 10^{-7} m$

$B = 3.5 \times 10^{-1} \, W/m^2$

$\theta = 90°$, $q = -1.6 \times 10^{-19} C$

$$\omega = \frac{qB}{m}$$

$$\omega = \frac{-1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$\omega = -6.147 \times 10^{10} \, rad/s$

(4c) the frequency at which the electron moves perpendicular to the direction of the uniform magnetic field, B is $-6.147 \times 10^{10} \, rad/s$

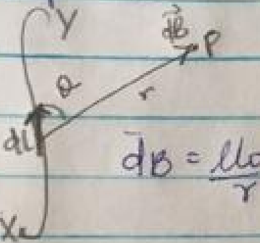
5) Biot Savart Law states that the magnetic field ($\vec{dB}$) at point P due to the small current element, Idl of current carrying conductor is

(i) directly proportional to dl

(ii) Inversely proportional to r^2

iii) directly proportional to $\sin\theta$

iv) Perpendicular to $\hat{r}$ and dl

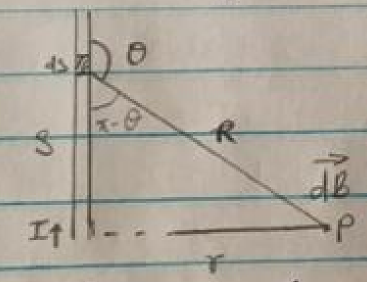


$$\vec{dB} = \frac{\mu_0}{r}\frac{I\,dl \times \hat{r}}{r^2}$$

6) Prove $B = \frac{\mu_0 I}{2\pi r}$

recall Biot Savart's Law

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \times \hat{r}}{r^2}$$



from above diagram, $\hat{r} = 1$, $\sin\theta = X$

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{R^2}$$

Note:

$\sin\theta = \sin(\pi - \theta)$

Using Pythagoras, $R^2 = s^2 + r^2$

$\therefore R = \sqrt{s^2 + r^2}$

using trig. $\sin(\pi - \theta) = \frac{r}{R} = \frac{r}{\sqrt{s^2+r^2}}$

assuming the wire runs infinitely

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\,ds}{R^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin(\pi-\theta)\,ds}{s^2+r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\frac{r}{\sqrt{s^2+r^2}}}{s^2+r^2}\,ds$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{r}{(s^2+r^2)^{3/2}}\,ds$$

recall special Integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

$\therefore$

$$B = \frac{\mu_0 I \cancel{r}}{4\pi}\left.\frac{1}{r^{\cancel{2}}}\frac{S}{(S^2+r^2)^{1/2}}\right]_{-\infty}^{\infty}$$

$$B = \frac{\mu_0 I}{4\pi r}\left[\frac{S}{(S^2+r^2)^{1/2}}\right]_{-\infty}^{\infty}$$

$$B = \frac{\mu_0 I}{4\pi r}\left(1-(-1)\right)$$

$$B = \frac{\mu_0 I}{\cancel{4}\pi r}\times \cancel{2}$$

$$B = \frac{\mu_0 I}{2\pi r}$$