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MECHATRONIC ENGINEERING

19/ENG05/052

1. A particle moves along a curve $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$, where t is time. Find its velocity.

$r = xi + yj + zk$

$r = 7t^2 i + (6t^2 - 4t)j + (t - 5)k$

$\frac{dr}{dt} = 14i + (12t - 4)j + k$

2. If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$, find $A \times (B \times C)$

$$B \times C = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$= i(9 - 4) - j(-6) + k(8)$

$= 5i + 6j + 8k$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j\begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$= i(16 + 24) - j(8 + 20) + k(6 - 10)$

$= 40i - 28j - 4k$

3. Given $R = (4\sin 3t)i + 4e^{3t}j + 7t^3 k$, find the integral of R with respect to t

$$\int R\,dt = \int (4\sin 3t)i\,dt + \int 4e^{3t}j\,dt + \int 7t^3 k\,dt$$

$$= 4\int \sin 3t\, i\,dt + 4\int e^{3t}j\,dt + \frac{7t^4}{4}$$

$\int \sin 3t \, dt$

$y = 3t$

$\frac{dy}{dt} = 3$

$dt = \frac{dy}{3}$

$\int \sin y \cdot \frac{dy}{3}$

$\frac{1}{3} \int \sin y \cdot dy$

$-\frac{1}{3}(\cos y)$

$-\frac{1}{3}\cos 3t$

$\therefore -\frac{1}{3}\cos 3t$

$\int R = -\frac{4}{3}\cos 3t + \frac{1}{3}e^{3t}$

$\int R = -\frac{4}{3}\cos 3t + \frac{1}{3}e^{3t} + \frac{7t^4}{4}$

(4) If $A = 2i + 2j - k$,
$B = 2i + j + 4k$,
$C = i + j + k$, find
$(A+C) \cdot (B-A)$

(5)

$A + C = (2+1)i + (2+1)j + (-1+1)k$

$A + C = 3i + 3j + 0$

$B - A = (2-2)i + (1-2)j + (4-(-1))k$

$B - A = -j + 5k$

$(A+C) \cdot (B-A) = (-0 - 3 + 0)$

$= -43$

Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$

$$R = xi + yj + zk$$

$$R = ti + t^2 j + t^3 k$$

$$\frac{dr}{dt} = i + 2tj + 3t^2 k$$

$$\frac{dr}{dt} = i + 2j + 3k$$

$$\left|\frac{dr}{dt}\right| = \sqrt{1 + 4 + 9}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{14}$$

$$T = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$$

$$\frac{i + 2j + 3k}{\sqrt{14}}$$