1 $\int \frac{(11-3x)}{x^2+2x-3}\,dx$

$\int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \left[\frac{A}{x+3} + \frac{B}{x-1}\right] dx$

$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$

$11-3x = A(x-1) + B(x+3)$

Let $x=1$

$11-3(1) = A(1-1) + B(1+3)$

$11-3 = 4B$

$4B = 8$

$B = 2$

Let $x = -3$

$11-3(-3) = A(-3-1) + B(-3+3)$

$11+9 = -4A$

$-4A = 20$

$A = -5$

$\therefore \frac{11-3x}{(x+3)(x-1)} = \frac{-5}{x+3} + \frac{2}{x-1}$

$\int \left[\frac{-5}{x+3} + \frac{2}{x-1}\right] dx$

$= -5\ln(x+3) + 2\ln(x-1) + c.$

2 $\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\, dx$

$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

let $x = 2$

$2(2)^2 - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$

$8 - 18 - 35 = B(3)(5)$

$15B = -45$

$B = -3$

let $x = -1$

$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$

$2 + 9 - 35 = A(-3)(2)$

$-6A = -24$

$A = 4$

let $x = -3$

$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$

$18 + 27 - 35 = C(-2)(-5)$

$10C = 10$

$C = 1$

$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3}$

$\int \left[\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \right] dx$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$

3. $\int \frac{1}{x^2+121}\,dx$

Recall $\int \frac{dx}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Where $a^2 = 121 \quad \therefore a = 11$

$\therefore \int \frac{1}{x^2+121}\,dx = \frac{1}{11}\tan^{-1}\frac{x}{11} + c.$