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MECHATRONICS ENGINEERING

MAT 104 PRACTICE QUESTIONS

① $\int \frac{11-3x}{(x-1)(x+3)}\,dx$

$y = \frac{11-3x}{(x-1)(x+3)}$

$= \frac{A}{x-1} + \frac{B}{x+3}$

$= \frac{(x-1)B + A(x+3)}{(x-1)(x+3)}$

$11-3x = (x+3)A + (x-1)B$

$11-3x = (A+B)x + 3A - B$

$11 = 3A - B$

$-3B = A + B$

$8 = 4A \;;\; A = 2$

$B = -5$

$\int \frac{2}{x-1}\,dx + \int \frac{-5}{(x+3)}\,dx$

$2\ln(x-1) + -5\ln(x+3) + C$

② $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} \;;\; y = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$= \frac{(x-2)(x+1)C + (x-2)(x+3)A + (x+1)(x+3)B}{(x-2)(x+1)(x+3)}$

$2x^2-9x-35 = (A+B+C)x^2 + (A+4B-C)x - 6A + 3B - 2C$

$A+B+C = 2 \;;\; A+4B-C = -9 \;;\; -6A+3B-2C = -35$

$A = 4 \qquad B = -3 \qquad C = 1$

$= \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$

$+4\ln(x+1) \quad -3\ln(x-2) + 2n(x+3) + C$

(3) $\int \frac{1}{x^2+a^2}$

$u = \frac{x}{a}$

$\frac{du}{dx} = \frac{1}{a}$

$dx = a\,du$

$\int \frac{1}{x^2+a^2} dx = \int \frac{1}{a(u^2+1)} du$

$= \frac{1}{a} \int \frac{1}{u^2+1} du$

$= \frac{1}{a} \tan^{-1} u$

$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$