

$$1) \int \frac{11-3x}{x^2+2x-3}$$

$$\int \frac{11-3x}{(x+3)(x-1)}$$

$$\frac{A}{x+3} + \frac{B}{x-1} = \frac{11-3x}{(x+3)(x-1)}$$

$$A(x-1) + B(x+3) = 11-3x$$

$$A(-3-1) = 11-3(-3) \quad \text{where } x = -3$$

$$-4A = 20$$

$$A = -5$$

$$\text{When } x = 1$$

$$B(1+3) = 11-3(1)$$

$$4B = 8$$

$$B = 2$$

$$\frac{-5}{x+3} + \frac{2}{x-1} = \int \frac{2dx}{x-1} - \int \frac{5dx}{x+3}$$

$$2 \ln|x-1| - 5 \ln|x+3| + C$$

$$2) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2-9x-35$$

$$A(x^2+3x-2x-6) + B(x^2+4x+3) + C(x^2-x-2) = 2x^2-9x-35$$

$$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$= 2x^2 - 9x - 35$$

$$(A+B+C)x^2 + (A+4B-C)x - 6A+3B-2C = 2x^2-9x-35$$

$$A+B+C = 2$$

$$A + B + C = 2$$

$$A + 4B - C = -9$$

$$-6A + 3B - 2C = -35$$

$$A = 2 - B - C$$

$$2 - B - C + 4B - C = -9$$

$$3B - 2C = -11 \quad \text{--- (1)}$$

$$-6A - 11 = -35$$

$$6A = 35 - 11$$

$$6A = 24$$

$$A = 4$$

$$2 = -B - C$$

$$B = -C - 2$$

$$3(-C - 2) - 2C = -11$$

$$-3C - 6 - 2C = -11$$

$$-5C = -5$$

$$C = 1$$

$$B = -1 - 2$$

$$B = -3$$

$$\int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{dx}{x+3}$$

$$4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$

$$3) \int \frac{dx}{x^2 + 121}$$

$$\int \frac{dx}{x^2 + 11^2}$$

$$x = 11 \tan \theta$$

$$\frac{dx}{d\theta} = 11 \sec^2 \theta$$

$d\theta$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2 + 11^2 = 11^2 \tan^2 \theta + 11^2$$
$$= 11^2 (\tan^2 \theta + 1)$$

$$\therefore \tan^2 \theta + 1 = \sec^2 \theta$$

$$x^2 + 11^2 = 11^2 \sec^2 \theta$$

$$\int \frac{11 \sec^2 \theta d\theta}{11^2 \sec^2 \theta}$$

$$\frac{1}{11} \int d\theta$$

$$\frac{1}{11} \theta + C$$

11

$$x = 11 \tan \theta$$

$$\frac{x}{11} = \tan \theta$$

11

$$\tan^{-1} \left(\frac{x}{11} \right) = \theta$$

$$\frac{1}{11} \tan^{-1} \left(\frac{x}{11} \right) + C$$

11