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DEPT: MECHATRONICS ENGINEERING

SUBJECT: MAT 102 ASSIGNMENT

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(1) $x = 7t^2$, $y = 6t^2$, $z = t - 5$ where $t = \text{time}$. find velocity.

$$\text{Velocity} = \frac{dr}{dt}$$

where $r = \text{displacement}$ and $r = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore r = 7t^2\hat{i} + 6t^2\hat{j} + (t-5)\hat{k}$$

$$\therefore \text{Velocity} = \frac{dr}{dt} = 14t\hat{i} + 12t\hat{j} + \hat{k}$$

$$\therefore \text{Velocity} = (14t)\hat{i} + (12t)\hat{j} + \hat{k}$$

(2) $A = \hat{i} + 2\hat{j} - 4\hat{k}$

$$B = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$C = 4\hat{j} - 3\hat{k}$$

find $A \times (B \times C)$

$$B \times C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= \hat{i} [9 - 4] - \hat{j} [-6 - 0] + \hat{k} [8 - 0]$$

$$B \times C = 5\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\begin{aligned} \therefore A \times (B \times C) &= \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix} \\ &= i \begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} \\ &= i [16 - (-24)] - j [8 - (-20)] + k [6 - 10] \end{aligned}$$

$$\therefore A \times (B \times C) = 40i - 28j - 4k$$

③ Given that $R = 4 \sin 3t i + 4e^{3t} j + 7t^3 k$

Find $\int R dt$

$$\therefore \int R dt = 4(-3 \cos 3t) i + 4\left(\frac{1}{3} e^{3t}\right) j + 7\left(\frac{t^4}{4}\right) k + C$$

$$\therefore \int R dt = -12 \cos 3t i + \frac{4}{3} e^{3t} j + \frac{7t^4}{4} k + C$$

④ $A = 7i + 2j - k$

$$B = 2i + j + 4k$$

$$C = i + j + k$$

find $(A+C) \cdot (B-A)$

$$\begin{aligned} A+C &= [7i + 2j - k] + [i + j + k] \\ &= (7i+i) + (2j+j) + (-k+k) \end{aligned}$$

$$A+C = 8i + 3j$$

$$\begin{aligned} B-A &= [2i + j + 4k] - [7i + 2j - k] \\ &= (2i-7i) + (j-2j) + [4k-(-k)] \end{aligned}$$

$$B-A = -5i - j + 5k$$

$$\begin{aligned} \therefore (A+C) \cdot (B-A) &= (8i + 3j) \cdot (-5i - j + 5k) \\ &= -40 - 3 \end{aligned}$$

$$(A+C) \cdot (B-A) = -43$$

(5) $x = t$, $y = t^2$, $z = t^3$ find unit tangent vector.

$$\text{Unit tangent vector } (\bar{T}) = \frac{dr/dt}{|dr/dt|}$$

$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

$$r = t\hat{i} + t^2\hat{j} + t^3\hat{k}$$

$$\frac{dr}{dt} = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

$$\frac{dr}{dt} = \hat{i} + 2(1)\hat{j} + 3(1)^2\hat{k}$$

$$\frac{dr}{dt} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\begin{aligned} \left| \frac{dr}{dt} \right| &= \sqrt{1^2 + 2^2 + 3^2} \\ &= \sqrt{1 + 4 + 9} \\ &= \sqrt{14} \end{aligned}$$

$$\left| \frac{dr}{dt} \right| = 3.74 \text{ units}$$

$$\therefore \bar{T} (\text{unit tangent vector}) = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{3.74}$$