

OLATUNJI IMOKEAYO OLUWASEYIFUNMI

MAs / MBS

MATs 102

19/mas01/333

Assignment 2

Examine whether or not these pair of lines are perpendicular to each other.

① $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

② $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

solution

① $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

let $A = y - 3x - 2 = 0$

$$\frac{dy}{dx} - 3 - 0 = 0$$

$$\frac{dy}{dx} - 3 = 0$$

$$\frac{dy}{dx} = 3$$

let $B = 3y + x + 9 = 0$

$$3\frac{dy}{dx} + 1 + 0 = 0$$

$$3\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = -1/3$$

A + B

∴ $y - 2x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

② $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

let $A = 3y - 4 = 2x + 3$

$$3 \frac{dy}{dx} - 0 = 2 + 0$$

$$3 \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

let $B = y - 5 = x + 6$

$$\frac{dy}{dx} - 0 = 1 + 0$$

$$\frac{dy}{dx} = 1$$

$A \neq B$

∴ $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ is not perpendicular.

③ Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution

$x^2 + y^2 + 3xy - 11 = 0$ at point $(1, 2)$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

Where $x = 1$ and $y = 2$

$$m = \frac{-(2(1) + 3(2))}{2(2) + 3(1)}$$

$$m = \frac{-(2 + 6)}{4 + 3} = -\frac{8}{7}$$

$$m = -8/7$$

Equation of the tangent to a curve

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -8/7(x - 1)$$

$$y - 2 = -8x/7 + 8/7$$

$$7x - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$7y + 8x - 22 = 0$$

Equation of the tangent of the curve

Equation of the normal to a curve

$$y - y_1 = -1/m(x - x_1)$$

$$y - 2 = -1/(-8/7)(x - 1)$$

$$y - 2 = 7/8(x - 1)$$

$$y - 2 = \frac{7x}{8} - 7/8$$

$$8y - 16 = 7x - 7$$

$$8y = 7x - 7 + 16$$

$$8y - 7x + 9 = 0$$

Equation of the ~~tangent~~^{normal} of the curve