

RECURSIVE

PROBLEMS

(Mechanical Engineering)

$$1) y = \frac{(x+1)(x-2)^{1/2}}{(x-1)(x+3)^{3/2}}$$

$$\ln y = \ln((x+1)) + \ln(\sqrt{x-2}) - \ln(x-1) - \ln((x+3)^{3/2})$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{(x+1)} + \frac{1}{\sqrt{x-2}} - \frac{1}{x-1} - \frac{3}{2(x+3)^{3/2}}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \frac{2}{2x-2} - \frac{3(x+3)^{3/2}}{2}$$

$$\frac{dy}{dx} = y \left[ \frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-2} - \frac{3}{2(x+3)^{3/2}} \right]$$

$$\frac{dy}{dx} = \frac{(x+1)(x-2)^{1/2}}{(x-1)(x+3)^{3/2}} \left[ \frac{2}{x+1} + \frac{1}{2x-2} - \frac{2}{2x-2} - \frac{3}{2(x+3)^{3/2}} \right]$$

$$2) y = \frac{3e^x \sin 2x}{x^{3/2}}$$

$$\ln y = \ln(3e^x) + \ln(\sin 2x) - \ln(x^{3/2})$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3e^x} \cdot 3e^x + \frac{1}{\sin 2x} \cdot 2\cos 2x - \frac{1}{x^{3/2}} \cdot \frac{3}{2}x^{1/2}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{3}{2}x^{-1/2}$$

$$\frac{dy}{dx} = y \left[ 1 + \frac{2\cos 2x}{\sin 2x} - \frac{3}{2}x^{-1/2} \right]$$

$$\frac{dy}{dx} = \frac{3e^x \sin 2x}{x^{3/2}} \left[ 1 + \frac{2\cos 2x}{\sin 2x} - \frac{3}{2x} \right]$$

Write on both sides of the paper

$$3. \int 9 \sec^2(3m+1) dm$$

$$= \int 9 \sec^2(3m+1) dm$$

$$u = 3m+1$$

$$\frac{du}{dm} = 3$$

$$du = 3 dm$$

$$dm = \frac{du}{3}$$

$$= \int 9 \sec^2(u) \frac{du}{3}$$

$$= \frac{9}{3} \int \sec^2(u) du$$

$$= 3 \tan(3m+1) + C$$

$$= 3 \tan(3m+1) + C$$

$$4. \int 2t (t^2-1)^{1/2} dt$$

$$u = \sqrt{t^2-1}$$

$$u^2 = t^2-1$$

$$2t^2 = u^2+1$$

$$t^2 = \frac{u^2+1}{2}$$

$$t = \frac{\sqrt{u^2+1}}{2}$$

$$\frac{dt}{du} = \frac{1}{2} \left( \frac{u^2+1}{2} \right)^{-1/2} \cdot \frac{2u}{2}$$

$$\frac{dt}{du} = \frac{u}{2 \left( \frac{u^2+1}{2} \right)^{1/2}}$$

$$dt = \frac{u du}{2 \left( \frac{u^2+1}{2} \right)^{1/2}}$$

$$\int 2 \left( \frac{u^2+1}{2} \right)^{1/2} \cdot \frac{u du}{2 \left( \frac{u^2+1}{2} \right)^{1/2}}$$

$$\frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \ln|x^2+1| + C$$

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

$$\int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} + C$$

$$\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-4}}{-4} + C = -\frac{1}{4x^4} + C$$

5  $\int \frac{2x}{\sqrt{4x^2-1}} dx$

$$u = \sqrt{4x^2-1}$$

$$\frac{d}{dx} \sqrt{4x^2-1} = \frac{8x}{2\sqrt{4x^2-1}} = \frac{4x}{\sqrt{4x^2-1}}$$

$$\frac{dx}{du} = \frac{1}{4} \sqrt{4x^2-1} \Rightarrow \frac{1}{4} \frac{1}{u}$$

$$\frac{dx}{du} = \frac{1}{4} \left( \frac{1}{u} \right)^{-1/2}$$

$$dx = \frac{1}{4} \frac{1}{\sqrt{u}} du = \frac{1}{4} u^{-1/2} du$$

$$\int \frac{2 \sqrt{4x^2-1}}{\sqrt{4x^2-1}} dx = \int \frac{2}{4} u^{-1/2} du = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \cdot 2u^{1/2} + C = \sqrt{4x^2-1} + C$$

$$\int \frac{1}{\sqrt{4x^2+1}} dx = \frac{1}{2} \int \frac{2}{\sqrt{4x^2+1}} dx$$

$$= \frac{1}{2} \ln|2x + \sqrt{4x^2+1}| + C$$