19/MHS01/424 MBBS MAT 104

1 $\int \frac{11-3x}{x^2+2x-3}\,dx$; Let $y = \int \frac{11-3x}{(x+3)(x-1)}\,dx$

By application of partial fractions,

$$\int \frac{11-3x}{x^2+2x-3}\,dx = \int \frac{A}{(x+3)} + \frac{B}{(x-1)}\,dx$$

$$\int \frac{11-3x}{(x+3)(x-1)} = \int \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}\,dx$$

$$\int \frac{11-3x}{(x+3)(x-1)} = \int \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}\,dx$$

By Comparing numerators

$$11-3x = A(x-1)+B(x+3)$$

$$11-3x = Ax-A+Bx+3B$$

$$11-3x = Ax+Bx-A+3B$$

∴ By comparing like terms

$$11 = -A+3B \quad \text{——①}$$

$$-3x = Ax+Bx$$

$$-3 = A+B \quad \text{——②}$$

$$① + ② = (11+(-3)) = (-A+A)+(3B+B)$$

$$8 = 4B$$

$$\therefore B = 8/4 = 2$$

Put $B = 2$ into eqn ①

$$\therefore 11 = -A+3(2)$$

$$11 = -A+6$$

$$\therefore A = 6-11 = -5$$

$$\therefore \int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \frac{-5}{(x+3)} + \frac{2}{(x-1)}\,dx$$

$$\int \frac{11-3x}{(x+3)(x-1)}\,dx = -5\int \frac{1}{x+3} + 2\int \frac{1}{x-1}\,dx$$

$= -5\ln(x+3) + 2\ln(x-1) + C$

2 $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

Let the integral of the above function be y

$\therefore y = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}\,dx$

$y = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{A(x-2)(x+3) + B(x+1)(x+3) + C((x+1)(x-2)}{(x+1)(x-2)(x+3)}$

By comparing numerators

$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

$2x^2-9x-35 = A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$

$2x^2-9x-35 = Ax^2+Ax-6A + Bx^2+4Bx+3B + Cx^2-Cx-2C$

By comparing like terms

$2x^2 = Ax^2+Bx^2+Cx^2$

$\therefore 2 = A+B+C$ —— ①

$-9x = Ax+4Bx-Cx$

$\therefore -9 = A+4B-C$ —— ②

$-35 = -6A+3B-2C$ —— ③

From eqn ① $2 = A+B+C$

$\therefore C = 2-A-B$ —— ④

Put ④ into ② and ③

② $= -9 = A+4B-(2-A-B)$

$-9 = A+4B-2+A+B$

$-9 = 2A+5B-2$

$\therefore 2A+5B = -7$ —— ⑤

③ $-35 = -6A+3B-2(2-A-B)$

$-35 = -6A+3B-4+2A+2B$

$-4A+5B = -31$ —— ⑥

⑤ − ⑥ $= 2A-(-4A)+(5B-5B) = -7-(-31)$

$6A = 24$

$\therefore A = 24/6 = 4$

Put $A = 4$ into eqn (5)

$2(4) + 5B = -7$

$8 + 5B = -7$

$\therefore 5B = -15$

$B = -3$

Put $A = 4$ and $B = -3$ into eqn (1)

$\therefore 2 = 4 - 3 + C$

$2 = 1 + C$

$C = 1$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{(x+1)} + \int \frac{-3}{(x-2)} + \int \frac{1}{(x+3)}\,dx$$

$$\therefore y = 4\int \frac{1}{x+1} - 3\int \frac{1}{x-2} + 1\int \frac{1}{x+3}\,dx$$

$$y = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$

3 $\int \frac{1}{x^2+121}$

$$\int \frac{1}{x^2+121} = \ln(x^2+121) + C$$