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1a) Three conditions for Couette flow:

- The velocity
- The flow rate
- The shear stress

1b) Four conditions to determine the nature of flow are;

- * Mean flow velocity (V)
- * The pipe diameter (D)
- * The density (ρ)
- * The fluid viscosity (μ).

2a) Difference b/w Aerofoil and hydrofoils are

Aerofoil

Hydrofoil

* Used when the fluid is gas.

* The foils (curves) are better than foils on a hydrofoil boat.

* Lift force is due to the angle of attack.

* Used when the fluid is liquid.

* The foils on a hydrofoil boat are smaller than the foils (curves) on an airplane.

* Lift force is due to the pressure on the bottom of the foil.

1b (ii) A liquid of 0.9 centipoise is filled between two horizontal plate of 10mm apart. If the upper plate moves at 1 m/s relative to the lower plate which is stationary and the pressure difference between the two sections 60m apart is 60 kN/m^2 , compute;



i) Velocity distribution.

ii) Discharge per unit width.

iii) Shear stress per unit width.

Solution

viscosity of the liquid $\mu = 0.9$; Centipoise = $0.001 \text{ poise} = 0.0009 \text{ (N) s/m}^2$

Distance b/w the plates, $b = 10 \text{ mm} = 0.01 \text{ m}$

Velocity of the upper plate, $u = 1 \text{ m s}^{-1}$

Pressure difference b/w the sections 6 cm apart $10^3 \text{ N/m}^2/\text{m}$

$$i) \quad u = \frac{y}{b} + \frac{1}{2\mu} \left(-\frac{\partial p}{\partial x} \right) (by - y^2)$$

$$= \frac{y}{0.01} + \frac{1}{2 \times 0.0009} (10^3) (0.01xy - y^2)$$

$$= y (100 + 5555.55 - 555555.55y)$$

$$\therefore u = y (5655.55 - 555555.55y)$$

ii) Discharge per unit width

$$Q = \int_0^b u dy$$

$$= \int_0^{0.01} (5655.55y - 555555.55y^2) dy$$

$$= \left[\frac{5655.55y^2}{2} - \frac{555555.55y^3}{3} \right]$$

$$= \left[\frac{5655.55 \times (0.01)^2}{2} - \frac{555555.55 \times (0.01)^3}{3} \right]$$

$$= 0.2827775 - 0.185185$$

$$= 0.0975 \text{ m}^3/\text{s}$$

iii) The shear stress at the upper plate.

$$\text{shear stress} = \mu \times \left(\frac{du}{dy} \right) = \mu \frac{dy}{dy} (5655.55y - 55555.55y^2)$$

$$= 0.0009 (5655.55 - 111111.1y)$$

∴ for the top plate; $y = 0.01 \text{ m}$

$$T_0 = 0.0009 (5655.55 - 111111.1 \times 0.01)$$

$$= 4.91 \text{ N/m}^2$$

2. Laminar flow of a liquid whose viscosity is 0.9 N s/m^2 and density of ~~1260~~ 1260 kg/m^3 occurs b/w a pair of parallel plates of extensive width inclined at an angle of 45° to the horizontal. The upper plate moved with a velocity of opposite to the fluid flow. Pressure gauges mounted at two points 1 m vertically apart on the upper ~~the~~ plate reads a pressure of 250 kN/m^2 & 80 kN/m^2 respectively.

Define a) Velocity & shear stress distribution b/w plates.

b) Max flow velocity

c) Shear stress on the upper plate.

Solution

$$\mu = 0.9 \text{ N s/m}^2 ; \rho = 1260 ; \text{specific gravity} = \frac{1260}{1000} = 1.26$$

$$b = 10 \text{ mm} = 0.01 \text{ m} ; u = 1.5 \text{ m/s} ; P_1 = 250 \text{ kN/m}^2 ; P_2 = 80 \text{ kN/m}^2$$

a) Velocity & shear stress distribution b/w the plates:

$$h_1 - h_2 = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right)$$

$$= \left(\frac{250 \times 10^3}{1.26 \times 9810} + 1 \right) - \left(\frac{80 \times 10^3}{1.26 \times 9810} + 0 \right)$$

$$= 21.225 - 6.47$$

$$= 14.755 \text{ mm} \text{ in } \sqrt{2} \text{ m} \text{ or } 1.414 \text{ m}$$

$$\frac{\partial h}{\partial x} = \frac{-14.755}{1.44} = -10.435$$

③

$$\frac{\partial p}{\partial x} = \omega \frac{\partial h}{\partial x} = (1.26 \times 9810 \times (-10.435))$$

$$= -128983 \text{ N/m}^2 \text{ or } -128.983 \text{ kN/m}^2$$

$$\therefore u = \frac{u \cdot y}{b} = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) (by - y^2)$$

$$= \frac{-1.5}{0.01} y - \frac{1}{2 \times 0.9} \times (-128.983 \times 10^3) (0.01y - y^2)$$

$$= -150y + 716.57y - 71657y^2$$

$$= 566.57y - (7.165 \times 10^4)y^2$$

The shear stress distribution is given by;

$$\tau = \mu \cdot \frac{v}{b} - \frac{1}{2} \frac{\partial p}{\partial x} (b - 2y)$$

$$= 0.9 \times (-1.5/0.01) - \frac{1}{2} \times (-128983) (0.01 - 2y)$$

$$= -135 + 644.92 - 128983y$$

$$= 509.92 - 128983y$$

$$= 509.92 - 1.289 \times 10^5 y$$

b) Maximum flow velocity, v_{max} ;

$$\text{For max velocity } \frac{dy}{dy} = 0$$

$$\frac{\partial}{\partial y} (566.57y - 71657y^2) = 0$$

$$566.57 - 143314y = 0$$

$$y = 566.57 / 143314$$

$$\therefore y = 3.95 \times 10^{-3} \text{ m}$$

$$v_{max} = 566.57 (3.95 \times 10^{-3}) - 71657 (3.95 \times 10^{-3})^2$$

$$= 2.238 - 1.118 \Rightarrow 1.12 \text{ m/s}$$

c) The shear stress on the upper plate;

$$\tau_{0.01} = 509.92 - (128983 \times 0.01)$$

$$= -780 \text{ N/m}^2$$

