UGWUJA OGBONNA 17/Eng09/072
Elect/Elect EEE 316

from $\frac{d^2 E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$

where $\rightarrow \frac{d^2 E_y}{dx^2} = \gamma^2 E_y$

$\gamma = \alpha + j\beta$

$E_y = \varepsilon_0 e^{-\gamma x} = \varepsilon_0 e^{-\alpha x} e^{-j\beta x}$

$\frac{\partial^2 E_y}{\partial x^2} = j\omega\mu\sigma E_y = \gamma^2 E_y$

$\gamma = \frac{\gamma^2}{\sqrt{j\omega\mu\sigma}} = \alpha + j\beta$

from $\sqrt{j} = \frac{1+1}{\sqrt{2}}$

We have; $\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\rightarrow \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ & $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$\therefore E_y = \varepsilon_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}} \times e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}}$

$\therefore$ We have; $\longrightarrow E_y = \varepsilon_0 e^{-x/\delta} e^{-jx/\delta}$

This indicates that the amplitude of the wave decreases esponentially as it generates a Conduction medium by a factor $\delta e^{-x/s}$

b Skin depth is the depth of penetration of a wave inside a conductor

$$\gamma = \sqrt{2/\omega\mu\sigma} \quad \text{and} \quad \omega = 2\pi f$$

$$= \sqrt{\frac{2}{2\pi f\mu r}} = \sqrt{\frac{1}{\pi f\mu r}}$$

$$\therefore \gamma = \sqrt{\frac{2}{\omega\mu r}} \quad \text{or} \quad \sqrt{\frac{1}{\pi f\mu r}}$$

c $f = 10MHz \simeq 1\times10^7 Hz$

$\gamma = 5.8\times10^7 s/m$, $\mu_r = 1$, $\mu_o = 1.257\times10^{-6}$

$$\delta = \sqrt{\frac{1}{\pi f\mu r}} = \frac{1}{\sqrt{\pi f\mu r}}$$

$\mu = \mu_r\times\mu_o = 1\times1.257\times10^{-6}$

1.257×10^{-6}

$$\delta = \frac{1}{\sqrt{\pi\times1\times10^7\times1.257\times10^{-6}\times5.8\times10^{7}}}$$

$\delta = 2.09\times10^{-5} m$

$b = 10mm$ $a = 3mm$, $\varepsilon_o = 8.85\times10^{-12}$

$= 0.01m$ $= 0.003$ $\mu_o = 1.257\times10^{-6}$

a Capacitance per meter

$$C = \frac{2\pi\varepsilon_o}{\log b/a}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log \frac{0.01}{0.03}}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$C = 3.84 \times 10^{-4}$ F/m

b

Inductance per meter b

$$L = \frac{\mu_0}{2\pi} \log_e b/a$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \cdot \log_e \frac{0.01}{0.03}$$

$$= \frac{1.257 \times 10^{-6}}{2\pi} \log 28.03$$

$L = 2.90 \times 10^{-7}$ H/m

Characteristics Impedance

$$Z_0 = \sqrt{L/C}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-4}}} \quad Z_0 = 86.90\Omega$$

Phase velocity

$$V_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.9 \times 10^{-7} \times 3.84 \times 10^{-11}}} = 29.10 \times 10^{7} V$$