Name: Cotterell Anna Obabari 19/SCI01/035

① A particle moves along a curve $x = 7t^2, y = 6t^2 - 4t$
$z = t - 5$, where t is time. find its velocity

position vector $r = x\hat{i} + y\hat{j} + z\hat{k}$

thus $r = (7t^2)\hat{i} + (6t^2 - 4t)\hat{j} + (t-5)\hat{k}$

velocity $= \frac{dr}{dt} = (14t)\hat{i} + (12t - 4)\hat{j} + \hat{k}$

② If $A = \hat{i} + 2\hat{j} - 4\hat{k}$, $B = 2\hat{i} - 3\hat{j} + \hat{k}$, $C = 4\hat{j} - 3\hat{k}$

find $A \times (B \times C)$

$$\bar{B} \times \bar{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$= \hat{i}(9-4) - \hat{j}(-6-0) + \hat{k}(8+0)$

$= 5\hat{i} + 6\hat{j} + 8\hat{k}$

$$A \times (B \times C) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$= \hat{i}(16+24) - \hat{j}(8-10) + \hat{k}(6-10)$

$= 40\hat{i} + 2\hat{j} - 4\hat{k}$

③ Given $R = 4 \sin 3t\, \hat{i} + 4e^{3t}\hat{j} + 7t^3 \hat{k}$, find the integral of R with respect to t

No 3 - Solution

$R = 4 \sin 3t \hat{i} + 4e^{3t} \hat{j} + 7t^3 \hat{k}$

$\int R \, dt = -\frac{4}{3} \cos 3t \hat{i} + \frac{4}{3} e^{3t} \hat{j} + \frac{7t^{3+1}}{3+1} \hat{k}$

$\int R \, dt = -\frac{4}{3} \cos 3t \hat{i} + \frac{4}{3} e^{3t} \hat{j} + \frac{7t^4}{4} \hat{k}$

④ If $A = 7\hat{i} + 2\hat{j} - \hat{k}$, $B = 2\hat{i} + \hat{j} + 4\hat{k}$, $C = \hat{i} + \hat{j} + \hat{k}$, find $(A + C) \cdot (B - A)$

$A + C = (7\hat{i} + 2\hat{j} - \hat{k}) + (\hat{i} + \hat{j} + \hat{k})$

$A + C = (7\hat{i} + \hat{i}) + (2\hat{j} + \hat{j}) + (-\hat{k} + \hat{k})$

$A + C = 8\hat{i} + 3\hat{j} + \cancel{0\hat{k}}$

$B - A = (2\hat{i} + \hat{j} + 4\hat{k}) - (7\hat{i} + 2\hat{j} - \hat{k})$

$B - A = (2\hat{i} - 7\hat{i}) + (\hat{j} - 2\hat{j}) + (4\hat{k} - \hat{k})$

$B - A = -5\hat{i} - \hat{j} + 3\hat{k}$

$(A + C) \cdot (B - A) = (8\hat{i} + 3\hat{j}) \cdot (-5\hat{i} - \hat{j} + 3\hat{k})$

$= -40 - 3 + 0$

$= -43$

⑤ Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$

Position vector $r = x\hat{i} + y\hat{j} + z\hat{k}$

thus $r = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

$$\frac{dr}{dt} = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{1^2 + (2t)^2 + (3t^2)^2}$$

where $t = 1$

$$= \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{1 + 4 + 9}$$

$$= \sqrt{14} = 3.7442$$

Hence $T = \dfrac{\hat{i} + 2\hat{j} + 3\hat{k}}{3.742}$