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181SCIOA1047

MAT 204

$$1. \quad X = \begin{pmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{pmatrix}$$

(i) Whether X is singular or non-singular

$$|X| = \begin{vmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{vmatrix}$$

$$|X| = 1 \begin{vmatrix} 7 & 6 \\ 5 & 3 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 9 & 3 \end{vmatrix} + 8 \begin{vmatrix} 4 & 7 \\ 9 & 5 \end{vmatrix}$$

$$|X| = 1(21 - 30) - 2(12 - 54) + 8(20 - 63)$$

$$|X| = -9 + 84 - 344$$

$$|X| = -269 \neq 0$$

\therefore ~~X~~ X is a non-singular matrix

(ii) Whether Y is singular or non-singular

$$|Y| = \begin{vmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{vmatrix}$$

$$|X| = 0 \begin{vmatrix} -7 & -1 \\ 1 & 9 \end{vmatrix} - 5 \begin{vmatrix} -3 & -1 \\ 2 & 9 \end{vmatrix} + 0 \begin{vmatrix} -3 & -7 \\ 2 & 1 \end{vmatrix}$$

$$|Y| = 0 - 5(-27 + 2) + 0$$

$$|Y| = 125 \neq 0$$

$\therefore Y$ is a non-singular matrix

(iii) Rank of Y

$$|Y| = 125 \neq 0$$

Hence, the rank of Y is 3

(iv) Whether $(X+Y)$ is singular or non

$$X+Y = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 0 \\ -3 & 7 & -1 \\ 2 & 1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 7 & 8 \\ 1 & 0 & 5 \\ 11 & 6 & 12 \end{bmatrix}$$

$$|X+Y| = \begin{vmatrix} 1 & 7 & 8 \\ 1 & 0 & 5 \\ 11 & 6 & 12 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & 5 \\ 6 & 12 \end{vmatrix} - 7 \begin{vmatrix} 1 & 5 \\ 11 & 12 \end{vmatrix} + 8 \begin{vmatrix} 1 & 0 \\ 11 & 6 \end{vmatrix}$$

$$= 1(0 - 30) - 7(12 - 55) + 8(6 - 0)$$

$$= -30 + 301 + 48$$

$$|x+y| = 319 \neq 0$$

$\therefore (x+y)$ is a non-singular matrix

② Whether $5Y$ is singular or non

$$5Y = 5 \begin{pmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{pmatrix}$$

$$5Y = \begin{pmatrix} 0 & 25 & 0 \\ -15 & -35 & -5 \\ 10 & 5 & 45 \end{pmatrix}$$

$$|5Y| = \begin{vmatrix} 0 & 25 & 0 \\ -15 & -35 & -5 \\ 10 & 5 & 45 \end{vmatrix}$$

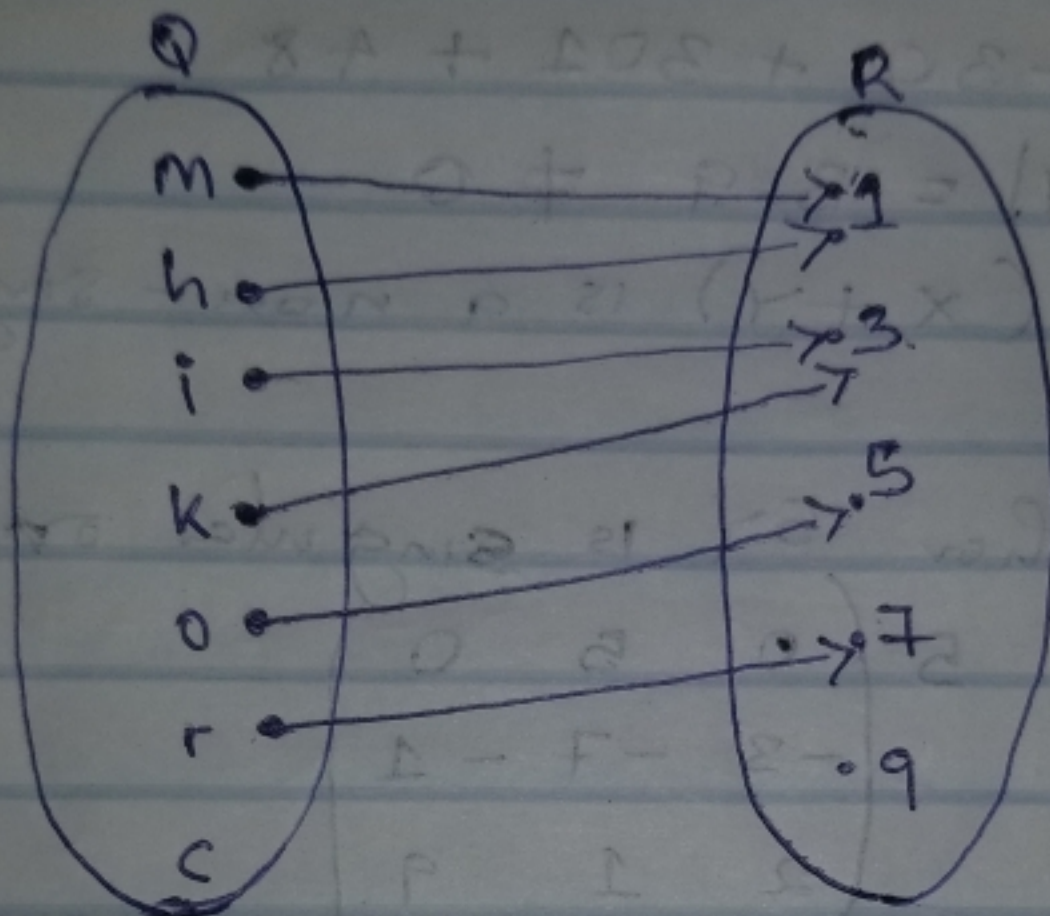
$$= 0 \begin{vmatrix} -35 & -5 \\ 5 & 45 \end{vmatrix} - 25 \begin{vmatrix} -15 & -5 \\ 10 & 45 \end{vmatrix} + 0 \begin{vmatrix} -15 & -35 \\ 10 & 5 \end{vmatrix}$$

$$= 0 - 25(-675 + 50) + 0$$

$$|5Y| = 15,625 \neq 0$$

$\therefore 5Y$ is a non-singular matrix

Q-2. $\mathbb{Q} \xrightarrow{T} \mathbb{R} = (0, 20 - 0) \cap \mathbb{R}$



Domain \mathbb{Q} Co-domain \mathbb{R}

$$T(m) = T(h) = 1$$

$$T(k) = T(i) = 3$$

$$T(o) = 5$$

$$T(r) = 7$$